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Abstract

Over the last several years, iPads have become increasingly popular in the classroom. The number of available apps that could be used in the mathematics classroom are countless, but some make better mathematical learning tools than others. This research presents a set of sixteen criteria that can be used to evaluate the potential of an iPad app to be an effective mathematical learning tool. A review of the existing literature on digital learning objects and on iPads and tablet PCs in the classroom is conducted. The evaluation instrument is presented, along with a discussion of each of the sixteen criteria. The instrument is then applied to seven apps designed for learning algebraic concepts. From this sample of seven apps, common themes are examined. Implications for educators, designers, and for future research are discussed.

Introduction

With the increasing popularity of tablet PCs and iPads, scores of K-12 and higher education institutions are adopting these devices to use in the classroom. However, one of the obstacles of using technologies like these to reshape the mathematics classroom is teachers being ill-equipped to determine how best to use these devices (Wilson, 2008). With the hundreds of applications available that could potentially be used in the classroom, not all of them can adequately enhance student learning and lead to concept development (Bos, 2009). In addition, teachers are unlikely to use an application in their classroom without some assurance of value and quality (Vargo, Nesbit, Belfer, & Archambault, 2003). Therefore, researchers, app designers, teachers and administrators need a way to evaluate whether an application would most likely be of greatest benefit to the learning of students. The purpose of this research is to fill this gap. To achieve this, we present a review of existing research on the use of mathematical learning objects and tablet PCs in the classroom, review previous criteria for effective digital learning objects, produce a set of criteria designed to evaluate interactive iPad applications for the learning of mathematics, and apply these criteria to evaluate seven applications for learning algebraic expressions and equations.

Digital Learning Objects

While the iPad itself is relatively new technology (the first edition was released in 2010), interactive digital tools and applications designed to explore mathematical concepts have been around for decades. For the purpose of this review, a "digital learning object" is defined as "any digital resource that can be reused to support learning" (Wiley, 2002). In recent years, many of these have taken the form of "applets" that appear on the internet. While there are some significant differences between web-based applets and iPad applications (the primary one being the touch interface, rather than typically a mouse and keyboard), they are, for the most part, very similar. Therefore, previous research on learning objects and criteria devised for evaluating them should provide a foundation on which to build a set of criteria for evaluating iPad applications that can be used in the mathematics classroom.

Recommendations from the National Council of Teachers of Mathematics ([NCTM], 2000) and the Common Core State Standards (National Governors Association Center for Best Practices, 2010) both call for the use of technology tools in the classroom. Additionally, there has been a call for a more student-centered, task-based learning environment (NCTM, 2000; National Research Council, 2001). Learning objects on the iPad can help educators achieve both of these goals. The iPad provides a safe student-centered environment, conducive to exploration and active learning, in which all students can participate (Cromack, 2008; Loch, Galligan, Hobohm & McDonald, 2011). In addition, teaching practices conducive to a student-centered, task-based learning environment have been found to be more prevalent in classrooms with tablet PCs, and teachers that use tablet PCs report that it is easier to create such environments (Cambre & Hawkes, 2004).

The field of mathematics contains a continuum of objects, ranging from the concrete (e.g., countable objects, visible patterns) to the abstract (e.g., formulas, variables, sets). Many mathematical objects can vary in how abstract they are perceived, depending on instruction and presentation. A digital learning object, in presenting a mathematical object on-screen, has the ability to make concrete objects more abstract and abstract objects more concrete. This allows students to alter mathematical objects that they previously viewed as concrete, enabling the student to extract properties of the object and see relationships between parts of the object or between different objects. For example, a slider that allows a student to adjust the coefficient in an equation allows the student to immediately see the resulting change in the graph (Figure 1). Conversely, a student can be presented an ordinarily abstract object, and be able to interact with it in tangible ways (Bos, 2009). In being able to directly manipulate such mathematical entities as variables, function rules, and equations, students obtain access to the tools of mathematics. These manipulations allow a student to link the specific and the general, and can lead to more complex understandings of objects on which the tools operate (Center for Implementing Technology in Education [CITEd], 2011; Clements, 2000; Heid, 2003; Lester, 2000; Moyer, Niezgoda, & Stanley 2005). According to Connell (2001), manipulating objects is the first and foundational step in object reification. In order to truly understand a mathematical concept, its mathematical object must be manipulated and conjectures made, then the object manipulated to test these conjectures, leading to further conjectures and problem-solving.

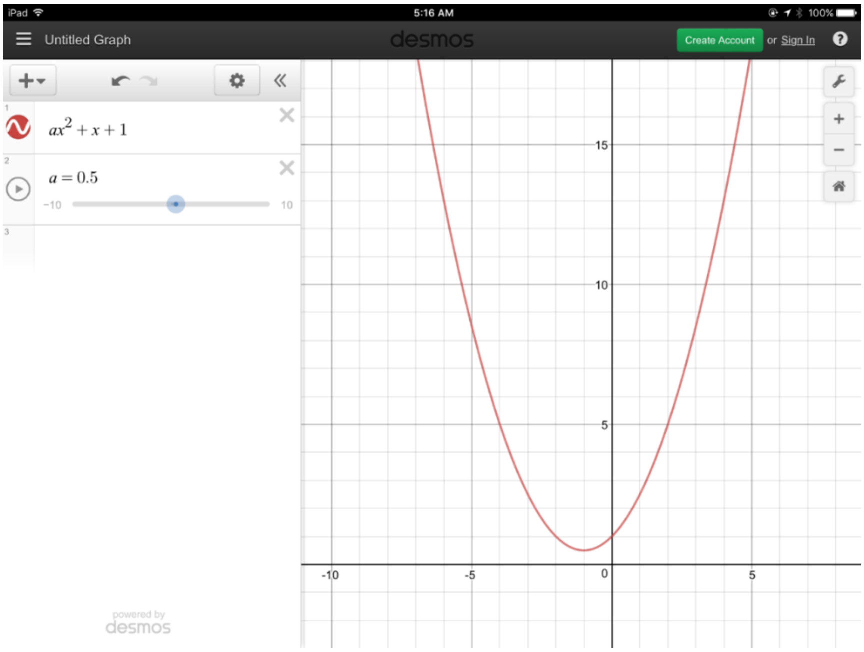

Figure 1. A slider to manipulate the graph

The Need for an Evaluation Metric

There are thousands of mathematics based apps available for the iPad, either for free or for a small price, with new apps continually being added. It is unlikely that a teacher would use these apps extensively in the classroom without some assurance of quality (Vargo et al., 2003). Thus, an effective evaluation metric could greatly reduce the search time for those teachers that only want to examine highly-rated apps (Koppi, Bogle, & Bogle, 2005). Especially for those apps with a cost to purchase, a lack of an evaluation metric could potentially discourage users, particularly those trying to limit costs, to try out an app that might otherwise be an effective teaching or learning tool (Downes, 2002). Teachers might then be compelled to use an unproven app, or create one of their own, rather than using one that has previously been an effective learning tool. Thus, one of the main benefits of learning objects, reusability, is compromised (Kay & Knaack, 2008a).

Choosing appropriate and pedagogically sound software is of great importance (Wilson, 2008, NCTM, 2000). Deciding which software is best for the classroom, however, requires an understanding of pedagogical principles that relate to technology use, something that many teachers lack (Gano, 2011; Wilson, 2008). Teachers must be aware of design principles and have a means by which they are able to discriminate between those apps which employ pedagogically sound design principles (Bos, 2009; Dick, 2008). Providing teachers with an evaluation metric based in pedagogical research allows them to do just this, and assures that their criteria for selecting apps is based on qualities that have been backed by research.

An evaluation metric can also be used prescriptively, by designers of apps, during the process of creating an app. There are many designers that aspire to make apps that are useful in the mathematics classroom, but not all of them will be aware of research that has been done on effective design. Without this knowledge, design decisions that these designers make (sometimes even subconsciously) can hinder the learning of students whom they wish to help. Giving these designers access to a research-backed set of criteria will allow them to incorporate elements into their apps that will enhance learning, and enable them to make more effective apps (Wiley 2000; Wiley 2002). This, in turn, can introduce multiple perspectives into the mathematics classroom, and give educators a wider range of effective apps to choose from (McCormick, Scrimshaw, Li, & Clifford, 2004).

Frameworks exist to analyze certain types of software for mathematics education, including, for example, digital textbooks and individualized learning programs (Choppin, Carson, Borys, Creosaletti, Gillis, 2014). However, there are significant differences between iPad apps and other digital learning objects which necessitate a need for an evaluation metric specific to iPad apps. There is currently a review system in place for iPad apps found in the iTunes App Store. Any user that owns an app can rate the app from 1-star to 5-stars, and optionally write an open-ended review, limited to approximately 5,500 characters. These ratings are then compiled, and a user browsing the store can see the average rating for an app, the distribution of 1-star to 5-star ratings, and read the reviews written by other users (Apple, 2017). But for educators selecting apps to use in the classroom, several shortcomings are present in the system. Typical reviews are open-ended, not addressing specific design characteristics that might enhance or reduce learning. Most reviewers review only a small selection of apps, making comparing reviews across different apps problematic. Reviews can vary widely in their content, ranging in length from a few words (or simply a numeric rating) to several paragraphs, and likely addressing different aspects of the app, again making comparison of apps difficult. A typically small sample of qualitative reviews makes it difficult to extrapolate experiences to entire populations. Finally, any claims in the reviews that the objects benefited learning are generally not statistically relevant, reliable, or research-backed (Kay & Knaack, 2007; Kay & Knaack, 2009). A standardized evaluation metric, based in research, would alleviate much of these problems.

Literature Review

Research on Digital Learning Objects

Research has shown that learning objects can enhance active learning and promote collaboration (McCormick et al., 2004; Parish 2004), both of which have been shown to improve student learning outcomes in many cases (Prince, 2004). Clements (2000) found that when using learning objects, students were more likely to cooperate and participate in positive discussion, increasing their social skills and promoting positive social interactions. Even when students disagreed, they tended to disagree about ideas, and were more likely to come to a successful resolution to their disagreements through discussion with their peers. In addition, students are more engaged when using learning objects (Kay & Knaack, 2007), and mathematical problems that would otherwise lack meaningful context can be transformed into tangible situations where students are eager to use their problem-solving skills to find solutions (McCormick et al., 2004; Reeuwijk, 2004).

Students have exhibited positive attitudes towards learning objects, both in anticipation of using them (CITEd, 2011), and after having used them. Students reported that they are easy to use (CITEd, 2011; Kenny, Andrews, Vignola, Schilz, & Covert, 1999; Reimer & Moyer, 2005), fun and motivating (CITEd, 2011; Kenny et al., 1999; Reeuwijk, 2004; Reimer & Moyer 2005; Vogel, Greenwood-Ericksen, Cannon-Bowers, & Bowers, 2006), more efficient than pen-and-paper methods (Reimer & Moyer, 2005), made them feel more confident about their math skills (Reeuwijk, 2004), and enhanced their learning (Reimer & Moyer, 2005; Riconscente, 2011). In addition, students reported that they appreciated the immediate feedback provided by the learning objects (Reeuwijk, 2004).

As computers became affordable and ubiquitous, attention from schools shifted from “learning to use computers to do math” to “using computers as an aid in a math lesson.” Early computer applications considered the computer to be another display medium and a source of drill-and-practice exercises. However, educators, especially those holding constructivist views of learning, began to oppose this approach and moved to make the computer a tool to aid in student-centered explorations of concepts and open-ended tasks (Durmuş & Karakirik, 2006). Learning objects can provide these learner-centered environments (Parrish, 2004, Loch et al., 2011). Such an environment allows and encourages the student to develop and explore their own mathematical ideas. A student can take control of his or her own learning, making and easily testing conjectures (Clements, 2000). In addition, the affordances of technology allows quicker testing of these ideas and more immediate feedback and apparent results, enabling the student to revisit their conjectures, analyze and revise them, and test them again (Bos, 2009; Laborde, 2007). Students are then more creative and get more self-esteem (Reeuwijk, 2004). Particularly for problems with multiple strategies and entry points, students can work at their own level of thinking, and thus individual differences in students are better addressed (Bos, 2009; Reeuwijk, 2004). When a student is feeling confused, learning objects can often provide hints and feedback, something that could otherwise not be done without teacher assistance (CITEd, 2011).

Research has suggested that such student-centered learning environments result in deeper understanding of mathematical concepts (Loch et al., 2011). Studies on learning objects suggest that this is indeed the case when learning objects are used in the mathematics classroom. Using learning objects to investigate problems and design solutions has been shown to improve a student's understanding of both the process and the content (Klopfer, Yoon, & Rivas, 2004). In addition, students develop their algebraic thinking and problem-solving skills when using learning objects (Reeuwijk, 2004). For example, while working with third graders using a “Pan Balance” applet, Polly (2011) found that students were able to analyze their solutions, evaluate both solutions and equations, and create new equations, simultaneously promoting deeper mathematical understanding as well as higher-order thinking skills. The visual and interactive features of learning objects allow a student to make connections between the visual and the symbolic, making the mathematics easier to understand and benefiting the student's learning (CITEd, 2011; Clements 2000; Reeuwijk, 2004; Suh & Moyer 2007). Because of this focus on the student, the large range of possible actions, and the helpful feedback that can be provided, Laborde (2007) reasons that learning objects are well-suited to help students develop the knowledge underlying optimal solving strategies.

In addition to promoting better understanding of mathematical concepts, learning objects can help a student communicate their understanding (or lack of understanding) to others by providing a window into a student's mathematical thinking. Difficulties and misconceptions that might otherwise get hidden in traditional methods are more readily revealed when using learning objects (Clements, 2000). Students have to translate their intuition into language and actions that the learning object understands, which in turn, causes the students to use formal mathematics language much more often when describing their ideas to their teachers and peers (Clements, 2000; Hoyles, Healy, & Sutherland, 2008). In addition, learning objects can be helpful for students with language difficulties, including English-language learners, who often have trouble communicating their ideas in the mathematics classroom. A learning object may help them clarify their thinking and demonstrate it to others. Also, several learning objects have support for multiple languages (CITEd, 2011).

The computing power of the technology behind the learning object also offers benefits for students' learning. Rather than the student being bogged down in arithmetic and procedures tangential to the intended learning goal, the computations can be passed off to the technology. This allows the student to focus on the mathematical concepts and models (Reeuwijk, 2004). In addition, the computing power allows for more immediate and accurate calculations, production of graphs, and other visible feedback in response to the user's actions, which has been shown to be valuable for mathematics learning (Forster 2006; Reeuwijk, 2004). The combination of a student-centered environment and the affordances that computing power provides ultimately leads to more efficient learning and cognitive material intake (Pange, 2003; Patsiomitou, 2008; Vogel et al., 2006).

There are practical advantages to learning objects, as well. Most web-based applets are free and iPad/iPhone applications generally range from free to a few dollars. The cost and effort to produce and distribute learning objects is lower than the alternatives, as well (Duval, Hodgkins, Rehat, & Robson, 2003), resulting in more perspectives and a larger variety of learning objects (McCormick et al., 2004). The content is more readily available, in that any classroom or home with an internet connection and a tool to run the learning object (e.g., computer, iDevice) can have access to any one of countless learning objects in minutes (Duval et al., 2003). Content, whether it be student work, custom problems and assignments, or learning objects themselves, can often be saved and easily shared between students and teachers, or between one teacher and another, either in nearby classrooms or around the world (Duval et al., 2003). This can lead to increased collaboration among teachers and propagation of new ideas about instruction (Parrish, 2004).

Learning objects can have a wide variety of applications in the mathematics classroom and can be used with a wide variety of students. Moyer-Packenham, Salkind, and Bolyard (2008) found that among K-8 teachers that used learning objects (specifically, virtual manipulatives), 45% used then in open-ended investigations and problem solving-activities. 37% used them as tools for skill solidification, 14% for introduction of new concepts, and the remaining 4% for other activities such as games, remediation aids, teacher models, and extension of concepts for students achieving above grade-level. In addition to these uses, Haughey and Muirhead (2005) claim that they can also be used to support new types of learning opportunities not available in a classroom environment, extend learning by providing new means for presenting curricular material, and illustrate concepts that are less easily explained through traditional teaching methods.

Learning objects can be effectively used with students of all grades, ranging from kindergarten (Moyer et al., 2005) to high school seniors (Bos, 2009; Kay, 2007). Additionally, learning objects have been shown to have an equally positive effect on males and females, both in regards to student performance (Kay & Knaack, 2008b) and attitude towards the learning object (Kay, 2007; Kay & Knaack, 2008b). And although self-reported computer comfort was shown to be correlated with reception of the learning objects (Kay, 2007), actual performance gains after using the objects was not correlated with computer comfort (Kay, 2007; Kay & Knaack, 2008b). Finally, learning objects have been shown to benefit students in honors classes (Reeuwijk, 2004), as well as those students classified as “low-achieving” (Bos, 2009).

Perhaps of most importance and interest, though, is that learning objects have been shown, in several studies, to increase student performance. In a study in Singapore (Looi & Kim, 2009), for example, a Secondary One class (approximately equivalent to seventh grade in U.S. schools) of 34 students worked for four weeks with a computer applet designed to assist students in constructing algebraic equations from word problems. Students' learning outcomes were measured with a post-test evaluating students' algebraic methods and workings. Results on this post-test were compared with those from a control classroom, which also had 34 students, closely matched in academic grades and previous scores on standardized tests. The post-test scores of the experimental group were significantly higher ($p<.001$) than the control group. The magnitude of the difference in scores was determined to be very large (eta-squared = 0.23).

In a study examining eighth and ninth grade algebra I classes across four secondary schools in the Netherlands, van Reeuwijk (2004) found that after integrating web applets into the instruction for a chapter, students scored better on the end-of-chapter tests than students without the applets in previous years. No information on significance or size of the improvement was provided from the study, however. In the United States, 95 low-achieving students in 19 classes across two districts participated in a study (Bos, 2009) examining the effects of computer-assisted instruction on a student's mathematical achievement when studying quadratic equations. The students were divided into a control group (47 groups in 9 classes) and an experimental group (48 students in 10 classes), each receiving 55 minutes of instructions per day for eight days. The experimental group's instruction took place in the computer lab, using the Texas Instrument InterActive environment, with lessons that focused on manipulating on-screen mathematical objects. The control group stayed in a traditional classroom, receiving instruction with more of an emphasis on lecture, taking scripted notes, and drill and practice. Post-test scores of the experimental group was significantly greater ($p<.001$) with a large effect size of .64.

Research on Tablets and iPads in Schools

Like desktop computers, early uses of tablet PCs and iPads in mathematics classrooms were primarily as convenience tools or instructional aids, rather than tools for exploring mathematical concepts (Hieb & Ralston, 2010; Loch et al., 2011; Oliver, 2005). In addition, much of the early research on tablets focused on university classrooms (e.g., see Fister & McCarthy, 2008; Oliver, 2005; Stickel, 2008). Over time, however, iPads and tablet PCs started appearing in K-12 schools, and some of the more recent research has taken place there.

Tablet PCs and iPads have been shown to increase student motivation across virtually all grades. In a study of 107 first- and second-graders with low socioeconomic status, all 57 students who solved mathematical tasks using iPad applications indicated positive motivation to use the applications (Segal, 2011). Crichton, Pegler, and White (2012) found that elementary and junior high students demonstrated “great enthusiasm” and described “great satisfaction” for using iPads, after a school year in which iPads were used extensively in the classroom. In a study examining fifth graders using an interactive iPad app to explore fractions, all 122 students reported that the app was fun and that they wanted to use the app again, and 95% reported that they thought their friends would enjoy it (Riconscente, 2011). Studies at the university level have indicated improved motivation (Galligan et al., 2012), increased student attention (Wise, Toto, & Lim, 2006), increased attendance and retention (Romney, 2010), very high participation rates in voluntary tablet activities (Vu, 2007), and increased participation in class (Reba & Weaver, 2007; Romney, 2010).

The use of Tablet PCs can lead to more efficient and effective learning. The use of regular, real-time formative assessment means that students and teachers can become more easily aware of what the student has mastered or of the student's misconceptions. This in turn, allows the teacher to focus on these misconceptions, rather than concepts already mastered by the student, leading to more efficient teaching and learning (Kowalski, Kowalski, & Hoover, 2007; Kowalski, Kowalski, & Gardner, 2009). Tutty and White (2006) argue that the tablet PC classroom environment is more effective than the traditional format, partly due to an increased emphasis on students' sense-making processes and on social aspects of learning. Students feel that they are able to learn mathematical concepts well in classes that use tablet PCs (Fister & McCarthy, 2008; Loch et al., 2011). Additional studies and reports suggest that the use of tablet PCs and tablet PC applications can result in an increased mathematical fluency (Attard & Northcote, 2011), students taking ownership of their learning (Fister & McCarthy, 2008), and an overall positive effect on student learning (Wise et al., 2006). When comparing students using tablets or iPads to their non-tablet-using counterparts, studies have found statistically significantly higher posttest scores for students using tablets or iPads starting in preschool (Schacter & Boaler, 2017), all the way to the university level (Fister & McCarthy, 2008).

One of the primary differences between a desktop PC and a tablet PC, besides portability, is the interface. Tablet PCs generally have a touch interface (either using a finger or a pen/stylus), while the most common interface for a desktop PC is a mouse and keyboard. Segal (2011) studied the differences between the effect these two interfaces had on young students' learning of mathematics. One hundred seven six- and seven-year-olds participated in completing two mathematical tasks, some completing the tasks on a tablet PC using the touch interface, and others performing identical tasks on a desktop with a mouse. In the first task (a counting and addition task), students using the touch interface spent significantly ($p < .001$) less time solving the problem (24% less, on average), and used more advanced strategies. In the second task (a number line estimation task), students again spent significantly ($p < .001$) less time solving the problem (32% less, on average). There was no significant difference in accuracy between students who used the tablet PCs and those who used desktop PCs.

Tablet PCs have practical advantages in a classroom, as well. The touch (or pen) interface allows symbolic and graphical information to be easily written electronically, and enables the teacher to adjust lectures in real-time, to explore different solution paths or in response to students' reactions or feedback (Loch & Donovan, 2006). One of the requirements for an effective Virtual Learning Environment according to the Shared Content Object Reference Model (SCORM) is a consistent standard for running and launching applications (Advanced Distributive Learning, 2009). The iPad in particular has an operating system which satisfies this – all applications are launched from the same place, and folders can be set up so that applications can be manually grouped according to course, topic, or any other desired grouping. Additional advantages include portability, a functional screen size, an abundance of apps, multiple routes for internet access, and multimedia, the combination of which can unleash numerous possibilities for learning and teaching (Gupta, 2010; Kinash, Brand, & Mathew, 2012; Loch et al., 2011).

Why Design Matters

Not all learning objects are effective at enhancing students' learning of mathematics, and using technology for the sake of novelty and interactivity is not enough to meet the increasing standards of today's educators and policy-makers (Bos, 2009). Past attempts to evaluate digital learning objects have shown that many of these digital learning objects tend to be lacking in key features that could enhance them beyond print materials (Choppin et al., 2014) There is a growing recognition that outcomes for learning are dependent on software design. The design of technology tools can drastically affect how students interact with the tools, and this, in turn, may influence students' understandings of content as well as their acquisition of problem-solving skills (e.g., Hoyles & Noss, 2003; Lee & Hollebrands, 2006). Learning objects, through their design, can also affect the available strategies a student can use, since they can afford or constrain certain actions (Underwood et al., 2005). The design of a learning object can be greatly influenced by the designers' epistemological and pedagogical beliefs about mathematics, problem solving, and teaching (Lee & Hollebrands, 2006). Effective design must be rooted in epistemological frameworks, and is possible only if the designer has a reflexive awareness of these frameworks (Bannan-Ritland, Dabbagh, & Murphy, 2000).

Previously Constructed Evaluation Models

Several authors, researchers, and organizations have devised models for evaluating learning objects, though not necessarily for the same reasons. Online learning object repositories, for example, may have criteria that must be met for inclusion into the repository (e.g., see Le@rning Federation, 2007). Other evaluation models attempt to provide a quantitative score to a set of existing objects in order to rank them or to assist educators wishing to use quality learning objects (e.g., see Bokhove & Drijvers, 2010; Multimedia Educational Resources for Learning and Online Teaching [MERLOT], 2013). Still others are meant as prescriptive standards intended for designers of learning objects (e.g., see Co-operative Learning Object Exchange [CLOE], 2004; Le@rning Federation, 2007). Ten such frameworks and evaluation models for digital learning objects (See Appendix A) were reviewed in order to construct a foundation for an instrument to evaluate mathematical iPad applications. The evaluation instruments reviewed come from a variety of sources. Some instruments were developed by a small team or an individual, while others were developed through collaboration with educators or experts around the world. Some were created through a systematic process of formation and refinement, while others did not have an established method for the creation of criteria. Some instruments were based on years of research, others were based on years of personal experience and anecdotal evidence. Some were specific to mathematics, while others were designed for digital learning objects in general. Despite all these differences, however, there were several design principles that appeared across numerous instruments (sometimes with slightly different phrasing). These common design principles – particularly the ones substantiated by other research – formed a set of criteria that was basis of the new evaluation instrument. These criteria were then adjusted, and new criteria were added, based on research particular to the use of iPads and other tablet PCs for learning.

The Evaluation Instrument

Using the 10 different evaluation models and other research on digital learning objects and tablet PCs in the classroom, an instrument to evaluate iPad applications for the learning of mathematics was created. The criteria developed in this instrument can be seen in Table 1. The criteria are grouped into four categories. In this section, an overall description of the instrument is provided, and then each criteria is further discussed with illustrative examples. In a later section, we illustrate how to apply this instrument to evaluate specific iPad apps.

Table 1. The criteria used in the evaluation instrument

Category	Criterion (each scored on a 5 point scale)
Interactions	Meaningfully interactive
	Natural actions on objects
	Visible consequences of actions
	Range of actions not limited by technology
	Necessitate thoughtful, deliberate actions
	Cognitively faithful interactions
Quality of Content	Mathematically faithful
	Cognitively demanding
	High-quality presentation
Feedback and Support	Provides relevant and timely feedback
	Provides scaffolding
	Provides opportunities for reflection
Usability	Easy to use
	Documentation is clear and easy to understand
	Interface is clear, consistent, and follows technology standards
	Usable by a variety of learners

Using the instrument, an app is given a score from 1 to 5 for each of the criteria, using the rubric in Table 2. The scores for each of the criteria within a category are then averaged to produce a score for each category. These four category scores are then averaged to produce a single overall score for the app. The overall score is calculated in this fashion so as to not give extra weight to categories with more criteria within them. What follows is a discussion and justification of each of the criteria

Table 2. Scoring rubric for the evaluation instrument

Score	**5**	The app meets the criteria very well. Any shortcomings are very minor and likely to not affect the student's learning.
	4	The app adequately meets the criteria most of the time, but with some flaws occasionally present or flaws that would only affect some users.
	3	There is significant room for improvement. There are a handful of flaws that have the potential to hinder students' learning.
	2	The criterion is not well-met. There are significant flaws present that are likely to hinder students' learning.
	1	The app does not meet the criterion at all, with major flaws, ones that could potentially prevent the app from being used as an effective learning tool.

The Interactions Category

Apps for learning mathematics should be *meaningfully interactive*. The app should consist of more than just a series of animations or static screens that the user simply watches and advances through (Lester, 2000). The user should have a choice of actions, and the on-screen objects should be transformed because of these actions. The reasons for this are twofold. First, allowing a student to manipulate on-screen objects has been shown to result in higher retention and increased problem-solving skills over students that watch non-interactive animations (Chan & Black, 2006). Second, these types of activities promote a student-centered, active learning environment, advocated by constructivist-learning theorists.

The mathematical actions that are performed should be *on objects* (Lester, 2000) and should be *natural* – that is, when an action is performed, the object should behave as expected, either because of the properties of the mathematical entity represented, or because of standards adopted. Note that this not only depends on the action being performed, but also on *how* the action is performed. Tversky, Morrison, and Betrancourt (2002) define the *congruence principle* as “the structure and content of the external representation should correspond to the desired structure and content of the internal representation” (p. 249). In other words, the way in which an action is physically performed (external) should match how the user perceives the mathematical action that is happening (internal). For example, if one was to rotate an on-screen figure by performing a rotation motion with their fingers, this would be a congruent action, while tapping on the figure would not be. Segal (2011) showed that in a touch-interface, students perform better on tasks that require congruent actions than those that require incongruent actions.

The *consequences of the user's actions should be clearly visible*. Perhaps this means momentarily highlighting an object before it changes, or emphasizing the link between the independent object that can be changed and the dependent object or objects that will change as a result. Changes that happen should be expected and noticeable. Meeting this criterion will help students better understand relationships (Underwood et al., 2005), a fundamental part of algebra and other mathematics.

The visibility of the consequences of one's actions contributes to the app's pedagogical fidelity, which Zbiek et al. (2007) define as “the extent to which teachers (as well as students) believe that a tool allows students to act mathematically in ways that correspond to the nature of mathematical learning that underlies a teachers practice” (p. 1187). It “is about allowing students to ‘do’ mathematics and not be distracted or limited by technical features” (Bos, 2009, p. 522).

Thus, as another requirement to achieve pedagogical fidelity, the *user's range of actions which can be performed on objects should not be limited by the technology.* It is students' reflections on these mathematical actions that lead to them making predictions and testing these predictions, ultimately leading to a greater understanding of the mathematical concepts and objects being represented on-screen (Dick, 2008). When actions that the user can perform are limited, the potential for students' learning is diminished. Therefore, the technology should allow the user to perform nearly action they could with pen-and-paper, and possibly more. An effective mathematics app should *necessitate thoughtful, deliberate actions*, rather than random ones. If a student completes a task by randomly performing actions, he or she is unlikely to learn the intended mathematical concepts or improve their problem-solving skills. In order to progress, students should need to carefully consider his or her actions.

Taken as a whole, the interactions afforded by the app should exemplify the app's *cognitive fidelity*. Cognitive fidelity refers to "how well the virtual tools reflect the user's cognitive actions and possible choices while using the tool in the virtual environment" (Moyer-Packenham et al., 2008, p. 204), or more succinctly, "whether a concept is better understood when the object is acted on" (Bos, 2009, p. 522). In the "real world", it is often acceptable for technological tools to arrive at "black box" solutions as efficiently as possible; however, in the classroom, learning apps should place a priority on illuminating mathematical thinking processes (Dick, 2008). For an example of how an algebra app might be cognitively unfaithful, consider the graph of the equation (i.e., a circle of radius 1 centered on the origin). If the app allows the user to scale the horizontal axis independently from the vertical axis, the circle seems to turn into an ellipse, with some points seemingly closer to the center than others. Particularly for a lower-level algebra student, this can cause confusion and obstruct the mathematical properties of a circle. The interactions afforded by the app should clearly convey the properties of the mathematical objects within it.

The Quality of Content Category

The mathematics in any app should be accurate; the app should possess *mathematical fidelity*. Mathematical fidelity is defined by Moyer-Packenham et al. (2008) as "the degree to which the mathematical object is faithful to the underlying mathematical properties of that object in the virtual environment" (p. 204). In the algebra classroom, these "mathematical objects" typically include such things as variables, real numbers, operations, equality, functions, and graphs. While mathematical accuracy may seem like an obvious goal to include in any learning app, it is not always easy to accomplish. Sometimes, it is the limits of the technology that are the obstacle: numerical precision, a pixelated display, the modeling of continuous structures with discrete ones. Other times, conscious design decisions are made to sacrifice mathematical fidelity for ease of use or to match user expectations. For example, Dick (2008) points to the interpretation of the user-entered expression sin2x. Many apps will take this to mean sin(2x), giving implicit multiplication a higher priority than application of a function. While this may be a convenience to the user, it introduces the unusual situation in which the implicit multiplication in sin2x is treated differently than the explicit multiplication in sin2 * x, where the application of the sine function would happen before the explicit multiplication (p. 335).

Apps for learning mathematics should be *cognitively demanding* (Bowers, Bezuk, & Aguilar, 2011; Dick & Hollebrands, 2011; Underwood et al., 2005). That is, they should require higher-level thinking rather than simple memorization or application of procedures. Note that perceived task difficulty is not necessarily correlated with cognitive demand. In algebra, for example, memorizing and using the quadratic formula may be difficult for some students, but an app that simply requires the application of the quadratic formula to several equations in the form $Ax^2 + Bx + C = 0$ is not cognitively demanding (Bowers et al., 2011). The app should evoke critical thinking from the user.

The *quality of the presentation should be high* (Kay & Knaack, 2008a; Nesbit et al., 2009; Underwood et al., 2005). This criterion is not the same as an attractiveness or an aesthetics criterion – the quality of the presentation should be judged based on how it enhances learning, not as one would judge the beauty of a piece of art. Multimedia in the app, including text, audio, graphics, video, and animations, should be clear and concise. Multiple types of multimedia should be incorporated in ways that enhance learning; in particular, the app should not be overloaded with needless or wordy text. Text and other media should be free of distracting typographical or technical errors. All components of the presentation should contribute to (or at the very least, not impede) student learning.

The Feedback and Support Category

The mathematics app should *provide relevant and timely feedback* (Kay & Knaack, 2008a; Le@rning Federation, 2003; MERLOT, 2013; Vargo et al., 2003). This feedback can take several forms. It can give the status of the problem being solved or of the mathematical objects being examined. It can be conceptual feedback, asking the user questions that cause them to reconsider their perceptions of the objects. It can be corrective feedback, letting the user know a mistake was made and guiding the user to correct the mistake. In any case, the feedback should provide formative assessment – that is, assessment *for* learning, rather than simply assessment *of* learning (Bokhove & Drijvers, 2010)

The mathematics app should *provide scaffolding* (Haughey & Muirhead, 2005; Le@rning Federation, 2003; MERLOT, 2013). The scaffolding should ensure that the user never feels like they are conceptually “stuck” or “lost” when it comes to advancing in the task. The scaffolding can be explicit, in the form of hints or guiding questions, or it can be implicit, by gradually increasing difficulty and building upon concepts. A good app will provide both. The mathematics app should *provide opportunities for reflection* (Clements, 2000; Dick & Hollebrands, 2011; Haughey & Muirhead, 2005; Underwood et al., 2005). This could take the form of a history of actions, or sense-making questions near the end of the task, or simply deliberate pauses throughout the task. Discussion could also be included regarding how the mathematics relates to the real world or to other fields of study.

The Usability Category

The mathematics app should be *easy to use* (Dick & Hollebrands, 2011; Kay & Knaack, 2008a; MERLOT, 2013). Most students should be able to successfully use the app without a lot of documentation, technical skills, or instruction. In other words, the app should be “pick-up-and-play”. When documentation or help files are present or necessary, *they should be clear and easy to understand* (CLOE, 2004; Haughey & Muirhead, 2005; Kay & Knaack, 2008a). Pictures, animations, or examples can be used to assist the user. Help files should be easy to navigate, with specific topics easy to find.

The *user-interface of the app should be clear, consistent, and follow technology standards*. A clear interface means that text should be large enough to be readable, buttons or other objects that can be clicked are clearly identified, and unnecessary information or dialogues are kept to a minimum. A consistent interface means that all or most pages have a similar look and feel and that text and visual components are always in the same location. Following technology standards means, for instance, that the user is expected to tap an object to select it or that icons for commonly used functions (save, pan, zoom) are instantly recognizable by the typical iPad or computer user.

The app should be *usable by a variety of learners* (Bokhove & Drijvers, 2010; CLOE, 2004; Haughey & Muirhead, 2005; Le@rning Federation, 2003; Underwood et al., 2005). Allowing multiple entry points, multiple approaches, and multiple solution strategies enables students of varying abilities and learning styles to express their own mathematical ideas, and engage with and complete the task. Multiple difficulty levels is another approach that can tailor the app to the learner's ability. The app can be effective for an even wider range of learners if multiple languages are supported, or if the app complies with accessibility standards for students with disabilities

Table 3. The apps reviewed

App	Developer	Version Reviewed	iTunes link
Hands-on Equations (Level 1)	Hands On Equations	3.9.5	https://itunes.apple.com/us/app/hands-on-equations-1-fun-way/id505948222
Algebra Touch	Regular Berry Software LLC	2.2.11	https://itunes.apple.com/us/app/algebra-touch/id384354262
Draognbox Algebra 5+	WeWantTo Know AS	1.3.2	https://itunes.apple.com/us/app/dragonbox+-algebra/id522069155
Numerosity: Play with Math! (Ch. 5)	Thoughtbox	1.1.1	https://itunes.apple.com/ie/app/numerosity-play-with-math!/id554637363
touchyMath	Joel Martinez	1.2.7	https://itunes.apple.com/us/app/touchymath/id388884486[3]
Cover Up	Steve Rhine	2.0	https://itunes.apple.com/us/app/cover-up/id541764631
Mathination – Equation Solver	Orion Math	1.1	https://itunes.apple.com/us/app/mathination-equation-solver/id396330842

Note. All apps were reviewed using the latest version available on the App Store as of April 25, 2017.

App Evaluations

After creation of the evaluation instrument, a sample of apps were chosen to apply the instrument to. The apps chosen focus on learning algebraic content, though the instrument can be applied to apps that focus on other mathematical content areas, as well.
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Selection of Apps

A multitude of apps were located via a variety of methods, including personal recommendation, web searches, and keyword searches in the App Store. Based on product descriptions and screenshots, the selection was filtered to include only those apps which were related to algebra and were educational (e.g., a quadratic equation solver that simply outputs the roots of an inputted function was not included). This resulted in approximately 25 apps that were downloaded for further inspection. After spending a few minutes with each app, this selection was again filtered to include only those apps that were interactive (e.g., digital versions of textbooks were not included) and intended to help users solve algebraic equations. This process resulted in seven apps, seen in Table 3, which serve as the sample to be discussed and evaluated using the evaluation instrument.

Evaluation Methods

Each of the seven apps were reviewed using the criteria in the evaluation instrument (see Table 1) and the rubric to rate each criterion as shown in Table 2. If an app had a pre-determined set of lessons, problems, or stages, the reviewer completed every one available. If an app allowed for user-generated problems, the reviewer created a purposeful sample of problems, including problems typical of everyday algebra curricula, as well as fringe cases that were more likely to result in mathematical errors from the app. Two apps were then randomly selected to be evaluated by an additional reviewer to assess the reliability of the instrument.

Table 4. Results of the App Evaluations

	Hands-on Equations	Algebra Touch	Dragonbox+ Algebra	Numerosity: Play with Math!	touchy Math	Cover Up	Mathination – Equation Solver	**Mean**
Meaningfully Interactive	4	5	5	5	5	3	5	**4.6**
Natural actions on objects	5	5	5	3	2	2	3	**3.6**
Visible consequences of actions	5	5	5	2	5	3	2	**3.9**
Range of actions not limited by technology	3	3	2	2	4	2	2	**2.6**
Necessitate thoughtful, deliberate actions	3	3	5	2	5	3	3	**3.4**
Cognitively faithful interactions	3	4	3	2	2	3	2	**2.7**
Interactions category overall score	**3.8**	**4.2**	**4.2**	**2.7**	**3.8**	**2.7**	**2.8**	**3.45**
Mathematically faithful	5	3	4	1	4	3	4	**3.4**
Cognitively demanding	4	4	3	2	4	2	3	**3.1**
High-quality presentation	4	5	5	2	5	2	5	**3.7**
Quality of content category overall score	**4.3**	**4.0**	**4.0**	**1.7**	**4.3**	**2.3**	**3.3**	**3.43**
Provides relevant and timely feedback	2	5	3	2	5	2	2	**3.0**
Provides scaffolding	3	4	3	3	4	3	4	**3.4**
Provides opportunities for reflection	2	2	2	1	4	3	3	**2.4**
Feedback and support category overall score	**2.3**	**3.7**	**2.7**	**2.0**	**4.3**	**2.7**	**3.0**	**2.95**
Easy to use	3	4	5	5	2	5	3	**3.9**
Documentation is clear and easy to understand	3	5	3	2	5	3	4	**3.6**
Interface is clear and consistent	5	5	5	4	3	3	3	**4.0**
Usable by a variety of learners	5	5	5	5	3	3	3	**4.1**
Usability category overall score	**4.0**	**4.8**	**4.5**	**4.0**	**3.3**	**3.5**	**3.3**	**3.89**
Total overall app score	**3.63**	**4.15**	**3.83**	**2.58**	**3.94**	**2.79**	**3.1**	**3.43**

Evaluation Results

The results of the evaluation of the seven algebra apps can be seen in Table 4. The total scores for the apps range from 2.58 to 4.15 (on a scale from 1 to 5). Figure 2 shows a screenshot of an app that scored highly. The interface is clean, and a simple animation allows the student to see the distributive property in action. The app also provides feedback to the student with both visual and audio cues, and guides the student through example problems during the interactive learning lessons. Figure 3 shows another app that scored highly. Students are guided step-by-step through these interactive lessons until they are able to solve problems on their own. Video tutorials are also given for many of the common actions. At any time, a student can pull up detailed information about the problem they are working on, showing all the steps completed so far.

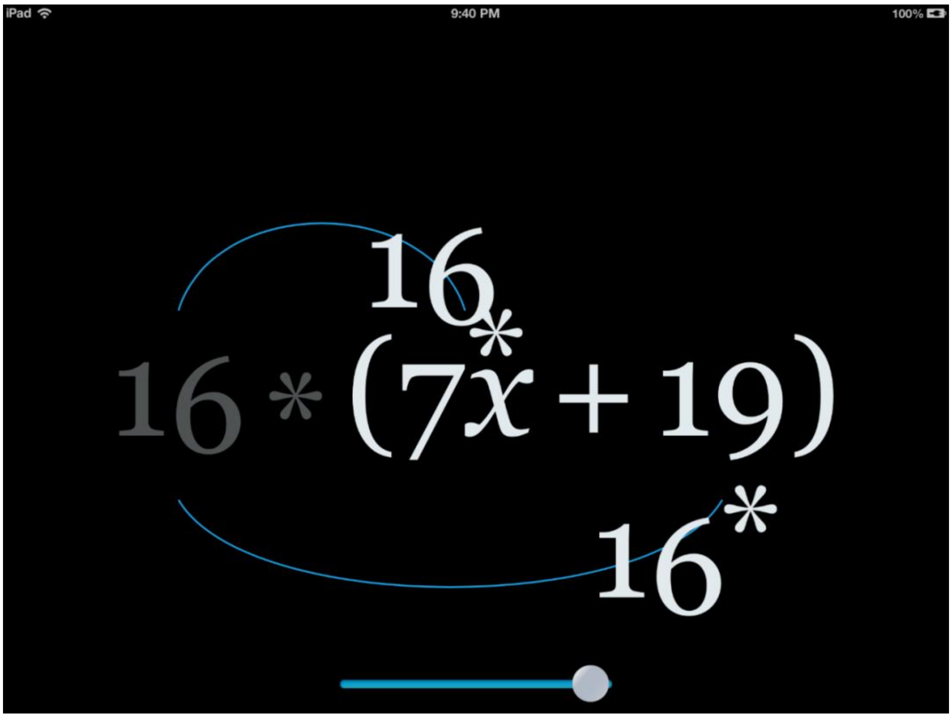

Figure 2. The distributive property in action

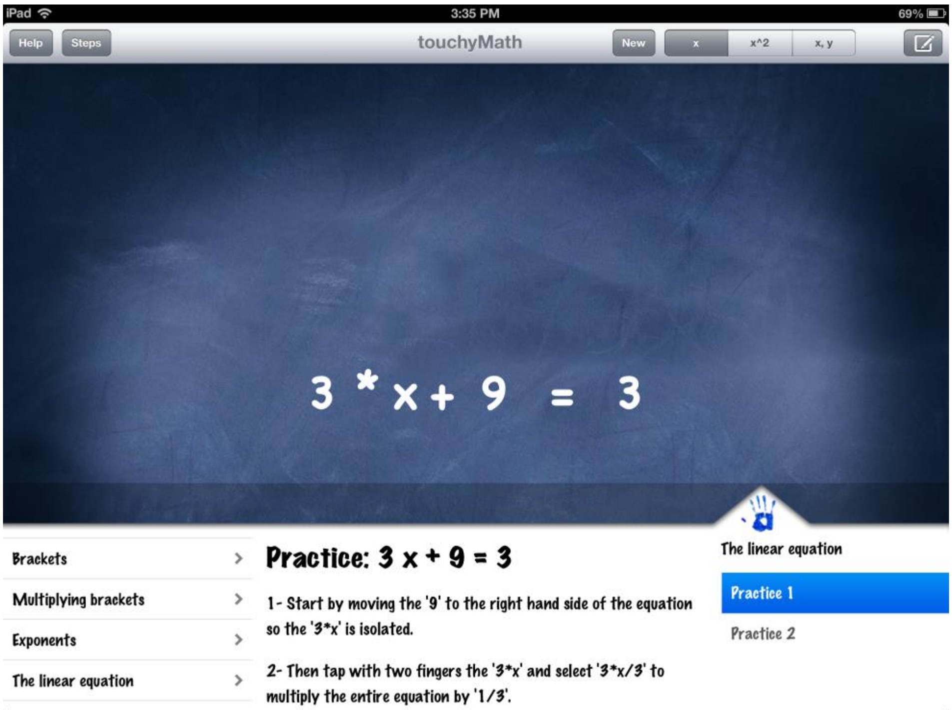

Figure 3. An interactive lesson on solving linear equations

Figure 4 shows a screenshot of an app that scored lower than others. The equation shown no longer makes mathematical sense. The timer and the score shown near the top of the screen are likely to cause students to rush through problems without taking the time to carefully consider their actions. The multiple-choice options allow students to progress simply by choosing each choice until the right answer is selected. Figure 5 shows another app that scored on the low end. The information displayed on the screen is often hard to comprehend. Long equations can run off the edge of the screen, with no way to pan or zoom. Feedback given to the student is limited to telling the student whether or not he/she has found the correct solution, without any indication of what error may have been made or how to correct it.

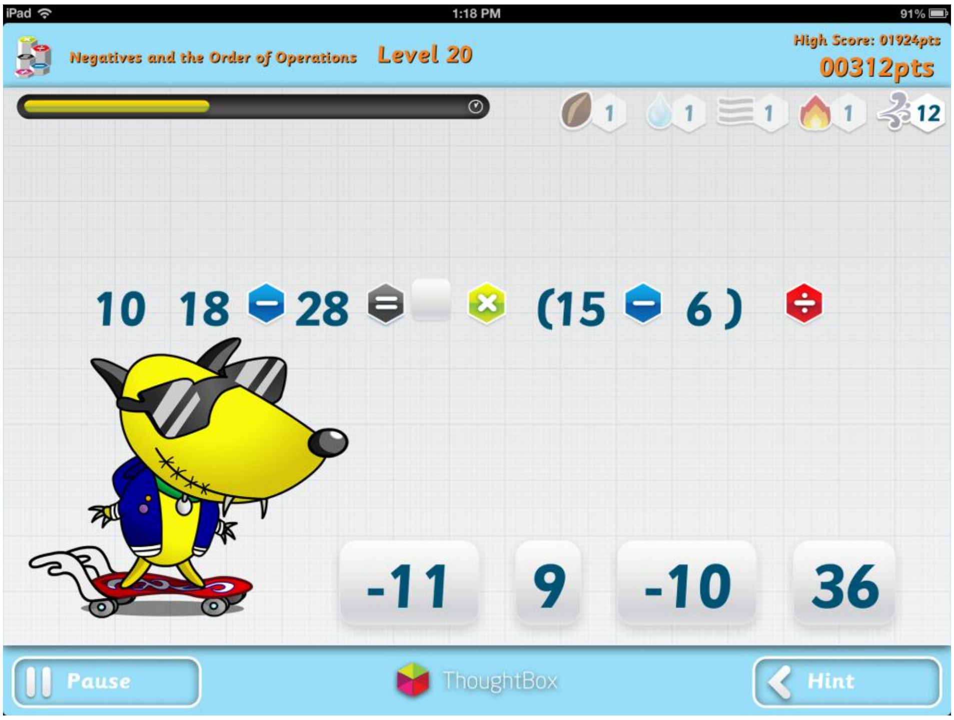

Figure 4. An equation that no longer makes sense

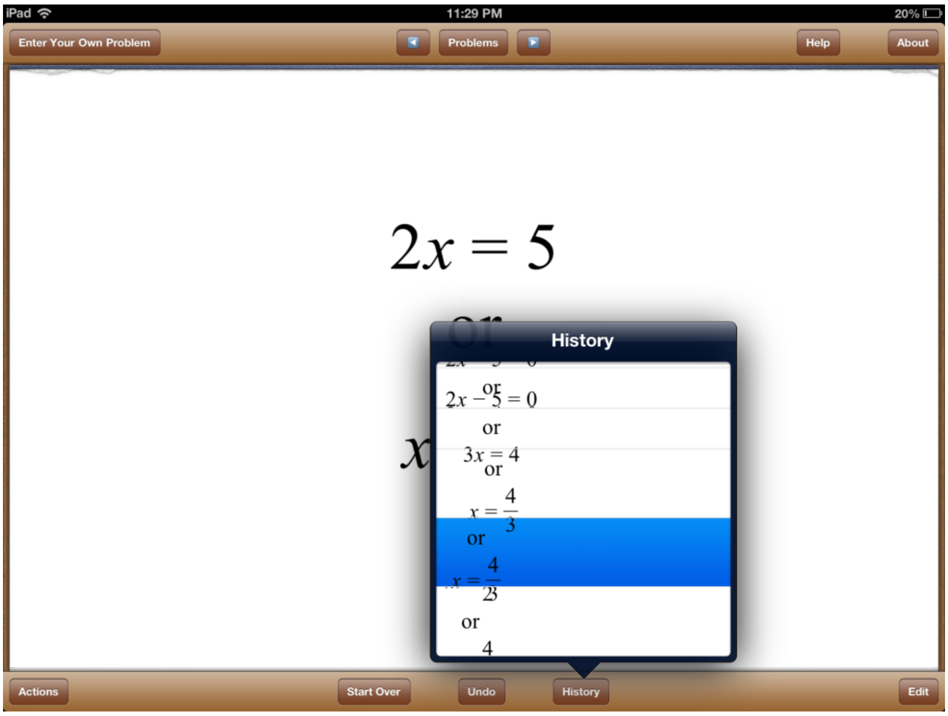

Figure 5. The history is difficult to comprehend

Certain criteria tended to see higher scores than others. The highest average score by far (4.57) is in the criteria *meaningfully interactive*. One of the steps in the process of selecting the apps to review, however, was filtering for a certain level of interactivity, so there is a selection bias influencing the scores in this criterion. After *meaningfully interactive*, the four criteria with the highest average scores are *usable by a variety of learners* (4.14), *interface is clear, clean, and follows technology standards* (4.00), *visual consequences of actions* (3.86), and *natural to use* (3.86). Three out of these four are criteria that make up the *Usability* category.

The lowest average scores, on the other hand, are for the criteria *provide opportunity for reflection* (2.43), *range of actions not limited by technology* (2.57), *cognitively faithful interactions* (2.71), and *provide relevant and timely feedback* (3.00). These are areas in which developers may want to devote extra attention to when developing future apps of this type.

Discussion and Implications

We can split the seven apps into two groups, based on overall score from the instrument. The four "high" apps – Algebra Touch, touchyMath, Dragonbox+ Algebra, and Hands-on Equations, with scores ranging from 3.63 to 4.15 – and the three "low" apps – Mathination, Cover Up, and Numerosity, with scores ranging from 2.58 to 3.10. The main source of differentiation between the two groups came from two categories: *interactions* and *quality of content*. The apps in the high group scored very well in these two categories, while apps in the low group scored poorly. These also happen to be the two categories in which an app that scored well in would be an app that would most likely help students overcome the common difficulties in algebra – by helping them understand the parts of an equation, and by promoting formal methods and precise notation.

Most of the apps reviewed in the high group were significantly lacking in a single area. Hands-on Equations and Dragonbox+ Algebra, for example, scored a 3.8 or higher in every category except for feedback and support, where they scored a 2.3 and a 2.7, respectively. touchyMath, on the other hand, received the highest rating among all the apps in the feedback and support category, but was tied for the low score with a 3.3 in the usability category. What this means, however, is that with improvements in just that one area, these apps could perhaps become ones that have great potential in the classroom.

The purpose of this instrument is to aid in the evaluation and selection of appropriate iPad apps that can potentially aid in the learning of mathematics. However, the selection of an app is only the first step. How these apps are used can have as large an impact on a student's learning as which app is used (Laborde, 2007; Polly, 2011). Another entire instrument could be created on how to plan and implement effective instruction using these apps. Literature on effective mathematics learning indicates, for example, that teachers should pose mathematically rich tasks (Laborde, 2007), focus on the math rather than the technology (Dick & Hollebrands, 2011), support collaborative environments in which students can communicate their ideas (Polly, 2011), intervene at critical times (Hoyles & Noss, 2003) and so on.

While the selection of an app is only one of many steps towards effective classroom teaching and learning, it is an important one that has significant impact on the rest. An app with meaningful interactions and a high quality of content, for example, makes it easier for teachers to design rich mathematical tasks around the app. An app that is easy to use and does not impose unnecessary constraints on the user allows the teacher to focus students' attention on the underlying mathematical concepts rather than on the technology. An app that provides relevant and timely feedback and scaffolds students' understanding allows students to maintain an autonomy over their learning while still allowing the teacher to provide additional support when needed. An effective app does not supplant the need for effective teaching; it can, however, make effective teaching easier.

Even if educators are provided with tools to easily find and select iPad applications for their mathematics classroom, there are a number of other factors that could prohibit the widespread use of these apps. Teachers are often unfamiliar with iPads and generally need training on how to effectively use them in the classroom (Crichton et al., 2012). Even with training, however, integrating new technology into the classroom is not an easy task for teachers. Instead of following a textbook, they must design tasks and worksheets, manage several different kinds of time in their classrooms, and find ways to relate paper-and-pencil techniques to technology-assisted ones (Polly, 2011). Goos and Bennison (2002) found that even when teachers work in technology-rich environments, have had ample training, and are encouraged to use technology by the administration, they often do not develop a proclivity to use the technology.

So, while there are many factors that can ultimately determine the benefit an iPad app will have on students' learning of mathematics, this instrument can assist designers in developing, and researchers and educators in choosing, apps that possess qualities that research suggests will promote students' learning and understanding. Our study did not examine how mathematics teachers choose to use iPad apps with their students. This is an important area of research that could add to the field's understanding of mathematics teachers' technological, pedagogical, and content knowledge (TPACK) as described by Niess (2005) and others.

References

Advanced Distributive Learning (2009). *SCORM 2004* (4th ed.). Retrieved from http://www.adlnet.gov/capabilities/scorm/scorm-2004-4th

Apple, Inc. (2017) iTunes [software] Available from http://www.apple.com/itunes/

Attard, C., & Northcote, M. (2011). Mathematics on the move: Using mobile technologies to support student learning (Part 1). *Australian Primary Mathematics Classroom*, *16*(4), 29-31.

Bannan-Ritland, B., Dabbagh, N., & Murphy, K. (2000). Learning object systems as constructivist learning environments: Related assumptions, theories, and applications. In D.A. Wiley (Ed.) *The Instructional Use of Learning Objects.* Bloomington, IN: Association for Educational Communications and Technology.

Bos, B. (2009). Virtual math objects with pedagogical, mathematical, and cognitive fidelity. *Computers in Human Behavior*, *25*(2), 521-528.

Bowers, J., Bezuk, N., & Aguilar, K. (2011). Adapting the mathematical task framework to design online didactic objects. *International Journal of Mathematical Education in Science and Technology, 42*(4), 481-495.

Cambre, M., & Hawkes, M. (2004). *Toys, tools, and teachers: The challenges of technology*. Lanham, MD: R&L Education.

Center for Implementing Technology in Education (2011). Learning mathematics with virtual manipulatives. Retrieved from http://www.cited.org/index.aspx?page_id=151

Chan, M. S., & Black, J. B. (2006). Direct-manipulation animation: Incorporating the haptic channel in the learning process to support middle school students in science learning and mental model acquisition. *Proceedings of the International Conference of the Learning Sciences.* Mahwah, NJ: Lawrence Erlbaum.

Choppin, J., Carsons, C., Bory, Z., Cerosaletti, C., & Gillis, R. (2014). A typology for analyzing digital curricula in mathematics education. *International Journal of Education in Mathematics, Science and Technology*, *2*(1).

Clements, D. H. (2000). From exercises and tasks to problems and projects: Unique contributions of computers to innovative mathematics education. *Journal of Mathematical Behavior 19*, 9– 47.

Connell, M. L. (2001). Actions on objects: A metaphor for technology-enhanced mathematics instruction. In D. Tooke & N. J. Henderson (Eds.), *Computers in the schools* (Vol. 17 (1&2), pp. 143–172)

Crichton, S., Pegler, K., & White, D. (2012) Personal devices in public settings: Lessons learned from an iPod touch / iPad project. *The Electronic Journal of e-Learning*, *10*(1), 23-31.

Cromack, J. (2008) Technology and learning-centered education: Research-based support for how the tablet PC embodies the seven principles of good practice in undergraduate education *Proceedings from the 38th ASEE/IEEE Frontiers in Education Conference*, T2A- 1–T2A-4. Saratoga Springs, NY: ASSE/IEEE

Dick, T. P. (2008). Keeping the faith: Fidelity in technological tools for mathematics education. In G. W. Blume & M. K. Heid (Eds.), *Research on Technology and the Teaching and Learning of Mathematics: Volume 2* (pp. 333-340) Charlotte, NC: Information Age Publishing.

Downes, S. (2002). Design and reusability of learning objects in an academic context: A new economy of education? *USDLA Journal, 17*(1).

Durmuş, S., & Karakirik, E. (2006). Virtual manipulatives in mathematics education: A theoretical framework. *The Turkish online journal of educational technology*, *5*, 117-123.

Duval, E., Hodgkins, W., Rehat, D., & Robson, R. (2003). Introduction. In E. Duval, W. Hodgins, D. Rehak, & R. Robson (Eds.), *Learning Objects 2003 Symposium: Lessons learned, questions asked.* Honolulu, HI: Ed-Media

Fister, K. R., & McCarthy, M. L. (2008). Mathematics instruction and the tablet PC. *International Journal of Mathematical Education in Science and Technology*, *39*(3), 285-292.

Forster, P. A. (2006). Assessing technology-based approaches for teaching and learning mathematics. *International Journal of Mathematical Education in Science and Technology*, *37*(2), 145-164.

Galligan, L., Loch, B., McDonald, C., & Taylor, J. A. (2010). The use of tablet and related technologies in mathematics teaching. *Australian Senior Mathematics Journal*, *24*(1), 38-51.

Gano, L. R., (2011). Fitting technology to the mathematics pedagogy: Its effect on students' achievement. *Journal of College Teaching & Learning, 8*(11), 29-38.

Goos, M., & Bennison, A. (2002). Building learning communities to support beginning teachers' use of technology. *Proceedings of the annual conference of the Australian Association for Research in Education*. Brisbane, AU: Australian Association for Research in Education.

Gupta, B., & Koo, Y. (2010). Applications of mobile learning in higher education: An empirical study. *International Journal of Information and Communication Technology Education (IJICTE), 6*(3), 75-87.

Heid, K. (2003). Theories for thinking about the use of CAS in teaching and learning mathematics. In J. T. Fey (Ed.), *Computer Algebra Systems in Secondary School Mathematics Education,* (pp. 33–52). Reston, VA: National Council of Teachers of Mathematics.

Hieb, J. L., & Ralston, P. A. (2010). Tablet PCs in engineering mathematics courses at the JB Speed School of Engineering. *International Journal of Mathematical Education in Science and Technology*, *41*(4), 487-500.

Hoyles, C., Healy, L., & Sutherland, R. (2008). Patterns of discussion between pupil pairs in computer and non- computer environments. *Journal of Computer Assisted Learning*, *7*(4), 210-228.

Hoyles, C., & Noss, R. (2003). What can digital technologies take from and bring to research in mathematics education? In A. J. Bishop, M. A. Clements, C. Keitel, J. Kilpatrick & F. Leung (Eds.), *Second International Handbook of Mathematics Education* (323-349). Dordrecht, the Netherlands: Kluwer Academic Publishers.

Kay, R. (2007). A systematic evaluation of learning objects for secondary school students. *Journal of Educational Technology Systems*, *35*(4), 411-448.

Kay, R. H., & Knaack, L. (2007). Evaluating the learning in learning objects. *Open Learning*, *22*(1), 5-28.

Kay, R. H., & Knaack, L. (2008b). A formative analysis of individual differences in the effectiveness of learning objects in secondary school. *Computers & Education*, *51*(3), 1304-1320.

Kay, R. H., & Knaack, L. (2009). Assessing learning, quality and engagement in learning objects: The Learning Object Evaluation Scale for Students (LOES-S). *Educational Technology Research and Development*, *57*(2), 147-168.

Kenny, R. F., Andrews, B. W., Vignola, M. V., Schilz, M. A., & Covert, J. (1999). Towards guidelines for the design of interactive multimedia instruction: Fostering the reflective decision-making of preservice teachers. *Journal of Technology and Teacher Education, 7*(1), 13–31.

Kinash, S., Brand, J., & Mathew, T. (2012). Challenging mobile learning discourse through research: Student perceptions of Blackboard Mobile Learn and iPads. *Australasian Journal of Educational Technology*, *28*(4), 639-655.

Klopfer, E., Yoon, S., & Rivas, L. (2004). Comparative analysis of Palm and wearable computers for participatory simulations. *Journal of Computer Assisted Learning, 20*, 347–359.

Koppi, T., Bogle, L. & Bogle, M. (2005). Learning objects, repositories, sharing and reusability. *Open Learning*, *20*(1), 83-91.

Kowalski, S. E., Kowalski, F. V., & Gardner, T. Q. (2009). Lessons learned when gathering real-time formative assessment in the university classroom using tablet PCs. In *Frontiers in Education Conference, 2009. FIE'09. 39th IEEE* (pp. 1-5). IEEE.

Kowalski, F., Kowalski, S., & Hoover, E. (2007). Using InkSurvey: A free web-based tool for open-ended questioning to promote active learning and real-time formative assessment of tablet pc-equipped engineering students. In *2007 ASEE Annual Conference*. Honolulu, HA: ASEE

Laborde, C. (2007). The role and uses of technologies in mathematics classrooms: Between challenge and modus vivendi. *Canadian Journal of Math, Science & Technology Education*, *7*(1), 68-92.

Lee, H. S., & Hollebrands, K. F. (2006). Students’ use of technological features while solving a mathematics problem. *Journal of Mathematical Behavior*, *25*(3), 252-266.

Loch, B., & Donovan, D. (2006). Progressive teaching of mathematics with tablet technology. *Journal of Instructional Science and Technology (e-JIST), 9*(2).

Loch, B., Galligan, L., Hobohm, C., & McDonald, C. (2011). Learner-centered mathematics and statistics education using netbook tablet PCs. *International Journal of Mathematical Education in Science and Technology*, *42*(7), 939-949.

Looi, C. K., & Lim, K. S. (2009). From bar diagrams to letter- symbolic algebra: a technology- enabled bridging. *Journal of Computer Assisted Learning*, *25*(4), 358-374.

McCormick, R., Scrimshaw, P., Li, N., & Clifford, C. (2004). CELEBRATE Evaluation report (version 2). Retrieved from http://www.eun.org/eun.org2/eun/Include_to_content/celebrate/file/Deliverable7_2EvaluationReport02Dec04.pdf

Moyer, P. S., Niezgoda, D., & Stanley, J. (2005). Young children’s use of virtual manipulatives and other forms of mathematical representations. *Technology-supported mathematics learning environments, 1*, 17.

Moyer-Packenham, P. S., Salkind, G., & Bolyard, J. J. (2008). Virtual manipulatives used by K-8 teachers for mathematics Instruction: Considering mathematical, cognitive, and pedagogical fidelity. *Contemporary Issues in Technology and Teacher Education*, *8*(3), 202.

National Council of Teachers of Mathematics (2000). *Principles and standards for school mathematics*. Reston, VA: National Council of Teachers of Mathematics.

National Governors Association Center for Best Practices, Council of Chief State School Officers (2010). *Common core state standards for mathematics*. Washington DC: National Governors Association Center for Best Practices, Council of Chief State School Officers.

National Research Council. *Adding It Up: Helping Children Learn Mathematics*. Washington, DC: The National Academies Press, 2001.

Niess, M. L. (2005). Preparing teachers to teach science and mathematics with technology:
Developing a technology pedagogical content knowledge. *Teaching and Teacher*
Education, 21(5), 509-523.

Olivier, W. (2005). Teaching Mathematics: Tablet PC technology adds a new dimension. In *Proceedings of the 8th International Conference on The Mathematics Education into the 21st Century Project: Reform, revolution and paradigm shifts in mathematics education* (pp. 176-181).

Pange, J. (2003). Teaching probabilities and statistics to preschool children. *Information Technology in Childhood Education Annual, 15*, 163–173.

Parrish, P. E. (2004). The trouble with learning objects. *Educational Technology Research and Development, 52*(1), 49-67.

Patsiomitou, S. (2008). Do geometrical constructions in a dynamic geometry environment affect students' algebraic expressions? *Proceedings of the Thirteenth Asian Technology Conference in Mathematics*. Retrieved from http://atcm.mathandtech.org/EP2008/papers_full/2412008_15001.pdf

Polly, D. (2011). Technology to develop algebraic reasoning. *Teaching Children Mathematics, 17*(8), 472-478.

Prince, M. (2004) Does active learning work? A review of the literature. *Journal of Engineering Education, 93,* 223–31.

Reba, M., & Weaver, B. (2007) Tablet PC-enabled active learning in mathematics: A first study. *First International Workshop on Pen-Based Learning Technologies* (pp. 1–6). Catania, Italy: IEEE.

Reeuwijk, M. van (2004). School algebra struggle, what about algebra computer games? In Niss M. (Ed.), *Proceedings of the Tenth International Congress on Mathematical Education*. Roskilde, Demark: Roskilde University

Reimer, K., & Moyer, P. S. (2005). Third-graders learn about fractions using virtual manipulatives: A classroom study. *Journal of Computers in Mathematics and Science Teaching*, *24*(1), 5-25.

Riconscente, M. (2011). Mobile learning game improves 5th graders' fractions knowledge and attitudes. Los Angeles: GameDesk Institute

Romney, C. A. (2010). Tablet PCs in undergraduate mathematics. In *40th ASEE/IEEE Frontiers in Education Conference*. Washington, DC.

Schacter, J., & Jo, B. (2017). Improving preschoolers' mathematics achievement with tablets: a randomized controlled trial. *Mathematics Education Research Journal*, 1-15.

Segal, A. (2011). Do gestural interfaces promote thinking? Embodied interaction: Congruent gestures and direct-touch promote performance in math. (Doctoral Dissertation). Retrieved from Columbia University Academic Commons (AC:P:10390)

Stickel, M. (2008). Effective use of tablet PCs for engineering mathematics education. In *Frontiers in Education Conference, 2008. FIE 2008. 38th Annual* (pp. S3J-7). IEEE

Suh, J., & Moyer, P. S. (2007). Developing students' representation fluency using virtual and physical algebra balances. *Journal of Computers in Mathematics and Science Teaching, 26*(2), 155-173.

Tutty, J., & White, B. (2006, January). Tablet classroom interactions. In *ACM International Conference Proceeding Series* (Vol. 165, pp. 229-233). Hobart, Australia: Australian Computer Society.

Tversky, B., Morrison, J. B., & Betrancourt, M. (2002) Animation: Can it facilitate? *International Journal of Human-Computer Studies, 57*, 247-262.

Vargo, J., Nesbit, J. C., Belfer, K. & Archambault, A. (2003). Learning object evaluation: Computer mediated collaboration and inter-rater reliability. *International Journal of Computers and Applications, 25*(3), 1-8.

Vogel, J. J., Greenwood-Ericksen, A., Cannon-Bowers, J., & Bowers, C. A. (2006). Using virtual reality with and without gaming attributes for academic achievement. *Journal of Research on Technology in Education*, *39*(1), 105-118.

Vu, C. (2007). Take a tablet. *Connections, the American Society for Engineering Education's Free E-newsletter, May*. Retrieved from http://www.asee.org/papers-and-publications/blogs-and-newsletters/connections/2007may.htm

Wiley, D. A. (2000). *Learning object design and sequencing theory*. (Unpublished doctoral dissertation). Brigham Young University, Provo, Utah. Available at http://opencontent.org/docs/dissertation.pdf

Wiley, D. A. (2002) Connecting learning objects to instructional design theory: A definition, a metaphor, and a taxonomy. In *The Instructional Use of Learning Objects* Bloomington, IN: Agency for Instructional Technology

Wilson, P. S. (2008). Teacher education: A conduit to the classroom. In G. W. Blume & M. K. Heid (Eds.), *Research on Technology and the Teaching and Learning of Mathematics: Vol. 2. Cases and Perspectives* (pp. 415–426). Charlotte, NC: Information Age.

Wise, J. C., Toto, R., & Lim, K. Y. (2006). Introducing tablet PCs: Initial results from the classroom. In *Frontiers in Education Conference, 36th Annual* (pp. 17-20). IEEE.

Zbiek, R. M., Heid, M. K., Blume, G. W., & Dick, T. P. (2007). Research on technology in mathematics education: The perspective of constructs. In F. Lester (Ed.), *Handbook of research on mathematics teaching and learning: Volume 2* (pp. 1169-1207). Charlotte, NC: Information Age Publishing.

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