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Abstract

This study investigates teachers' perspectives on the use of a mathematical argumentation teaching strategy in elementary mathematics in which students disprove mathematical statements they already know to be false. Mathematical argumentation is a process through which students develop an argument about a mathematical concept and rationalize its truth or untruth through mathematical reasoning but is if often underused in mathematics. In this study we focused on a argumentation tasks which involved providing students with number statements which they already knew to be false, and inviting them to argue, using visuals, numeric notation, and/or written explanations, why it was false. Through practical action research, seven teachers from two different schools implemented this approach in their mathematics programs to one hundred and thirty-one students over the course of five months. Findings indicate this approach was easy to implement, improved student engagement, supported learners who struggled and deepened students' mathematical knowledge. We believe that this approach can be used as a precursor to more formal proofs and provide more access for teachers and students in exploring mathematical proofs in elementary classrooms.

Introduction

Mathematical argumentation is a process through which students develop an argument about a mathematical concept and rationalize its truth or untruth through mathematical reasoning (Stylianides et al., 2013). For many, mathematical argumentation acts as a precursor to formal proof (Marks Krpan & Sahmbi, 2020), a fundamental component of the discipline of mathematics. In spite of its importance in mathematics, and though there is often an expectation that students are familiar with or ready to engage with proofs in secondary and post-secondary mathematics, students are seldom introduced to this way of mathematical thinking in elementary school (Stylianides, 2007b), or mathematical argumentation. This inexperience often contributes to the challenges students face when learning about proofs in secondary and post-secondary mathematics (Buchbinder & Zaslavsky, 2007; Yu et al., 2004). Further, research has shown that mathematical argumentation can improve collective understandings of mathematical concepts (Forman et al., 1998), yet mathematical argumentation is underutilized in most elementary classrooms. Researchers surmise that implementing mathematical argumentation or proof can be challenging for elementary teachers for a variety of reasons including concerns around limited mathematical content knowledge (Stylianides et al., 2013); uncertainty about students' abilities to grapple with the idea of proving (Stylianides & Ball, 2008); and limited time to establish an appropriate classroom environment conducive to effective integration of argumentation pedagogies (Makar et al., 2015). Further, there is limited research focused on the elementary context that examines student mathematical argumentation involving false propositions and the analysis of counter examples (Lee, 2016).

Given the importance of mathematical argumentation and proof-building in mathematics, we sought to pursue a study focused on elementary teachers implementing mathematical argumentation in their classrooms and explored their views on the impact of the argumentation tasks on student learning and strategies they found helpful for implementation. Teachers are critical players in the implementation of argumentation (Conner, 2022). Their perspectives on argumentation as a tool and as a tenet of mathematics impacts how students engage with argumentation in the classroom (Conner & Singletary, 2021; Gomez Marchant et al., 2022). As such, this study aimed to explore teachers' perspectives on using mathematical argumentation in primary classrooms. More specifically, we asked two questions: *1) How do teachers perceive the use of mathematical argumentation in elementary classrooms;* and *2) How do teachers perceive the impact of mathematical argumentation on student success in elementary mathematics?*

Perspectives

Proof and argumentation are fundamental components of the mathematics discipline that are related, yet distinct. Though there is considerable debate over exact definitions, Stylianides (2007a) explains that,

> Proof is a mathematical argument, a connected sequence of assertions for or against a mathematical claim, with the following characteristics: it uses statements accepted by the classroom community. . . it employs forms of reasoning that are valid. . . and it is communicated with forms of expression that are appropriate. (p. 281)

Relationally, Rumsey and Langrall (2016) posit that argumentation is "a process of dynamic social discourse for discovering new mathematical ideas and convincing others that a claim is true" (p. 414). Argumentation in mathematics is a process through which students may develop an argument about a mathematical concept and rationalize its truth or untruth through mathematical reasoning (Stylianides et al., 2013). Often, students engage in argumentation through classroom discussion (Marks Krpan, 2018; Marks Krpan & Sahmbi, 2020) where they work to convince their peers and/or build off of peers' ideas to develop their mathematical understanding (Knudsen et al., 2014), making it a social, communication-based activity (Marks Krpan, 2018). Thus, argumentation can play a dual role in the classroom of developing student understanding of mathematical concepts as well as existing on its own as a foundation of mathematical thinking (Staples & Newton, 2016).

Some researchers have found that the benefits of argumentation in mathematics classrooms extends beyond enhancing content understanding (Civil & Hunter, 2015; Rumsey & Langrall, 2016). Civil and Hunter (2015) suggest that argumentation, when taught through cultural lenses, "is a powerful reasoning tool which allows participants to elaborate and justify mathematical concepts and develop an understanding of opposing perspectives" (p. 297). Further, argumentation allows for student collaboration and personal agency in that "mathematical authority and ownership shift from the textbook or teacher to the community of learners" (Rumsey & Langrall, 2016, p. 414). Hence, argumentation as a tool and way of thinking in mathematics holds promise as a means of decentering the teacher and honouring students more readily in the classroom.

While both proof and argumentation are foundational in mathematics (Hanna & de Villiers, 2008; Stylianides, 2007a), students do not tend to engage with formal proof until secondary or post-secondary mathematics (Ball et al., 2002; Campbell et al., 2020; Stylianides, 2007b). This is particularly problematic, as this disconnect results in an abrupt shift from no proof to an expectation of engaging with formalized proof in secondary or post-secondary mathematics, and this transition can be challenging for students (Stylianides, 2007). Thus, many researchers argue that argumentation should be introduced to students in elementary grades to ease the transition (e.g., Ball et al., 2002; Bieda et al., 2014; Hanna & de Villiers, 2008) and to better support "coherence in students' learning of mathematical processes, such as reasoning and proving" (Bieda et al., 2014, p. 78). Indeed, a call for emphasis on reasoning and proving has been brought forward for years in curriculum documents (e.g., Ministry of Education, 2020) and through national mathematics organizations, alike (NCTM, 2000; NCTM, 2009). Though some may worry that elementary students are not ready to engage with proof and argumentation, Rumsey (2013) notes that "even without prior instruction emphasizing argumentation" (p. 4-127) children as young as the second grade are able to draw on existing knowledge to develop cogent arguments.

Mathematical argumentation in elementary years complements inquiry-based classrooms in which students explore mathematical conjectures and debate mathematical arguments of their peers (Goos, 2004). In an inquiry-based classroom, argumentation enables students to explore ideas and conjectures and determine if they make sense (Brown, 2017). Makar et al. (2015) describe how argumentation in elementary classrooms enables students to compare and justify ideas with each other and share these notions with the whole class for validation. The authors stress that infusing argumentation into inquiry supports cognitive engagement with mathematics and "emphasizes the negotiation and the discursive practices of inquiry" (Makar et al., 2015, p. 1107).

Argumentation is critical for making meaning of mathematics (Schwarz et al., 2010) and developing conceptual understandings in the discipline (Whitenack & Yackel, 2002). Though research has shown that the use of argumentation in mathematics can improve collective understanding of mathematical concepts (Forman et al., 1998), it is still not a widely understood or used pedagogy. Indeed, the role of the teacher in effectively implementing argumentation in the classroom is pivotal (Ayalon, 2019; Ayalon & Even, 2014; Knudsen et al., 2014; Solar et al., 2021; Stylianides, 2007a). Yet, implementing mathematical argumentation or proof can be particularly challenging for elementary teachers for many reasons, including their own varying levels of mathematical content knowledge (Stylianides et al., 2013); uncertainty about students' abilities to grapple with the idea of proving (Stylianides & Ball, 2008); and limited time to establish an appropriate classroom environment for the integration of argumentation pedagogies (Makar et al., 2015).

Bieda (2010) notes that "teachers' knowledge of proof and their beliefs about teaching proof may also constrain their ability to teach proof effectively" (p. 352). How teachers conceive of proof and argumentation influences not just how proof is taught, but what they deem is appropriate for their students (Stylianides, 2007a), and hence, if teachers' understanding of proof is limited, student interaction with proof will be limited, too. Further, because the focus of argumentation is for *students* to develop their reasoning and understanding, teachers must also be familiar with ways of facilitating effective mathematical discourse, which can be a major challenge for some (Ayalon & Even, 2014). In recent years, there has been literature exploring student teacher (Conner & Singletary, 2021) and teacher perspectives and beliefs (e.g., Gomez Marchant et al., 2022) on the use of argumentation and the subsequent impact that this has on the classroom. While these studies highlight the importance of the teacher in implementing argumentation effectively and engaging students in this aspect of mathematics, there continues to be limited work done specifically in the field of elementary mathematics.

For the purposes of this research, we consider the use of argumentation in elementary mathematics as a precursor to formal proof, that is, a process where one can conclusively *prove* and generalize mathematics. Even though typical argumentation may lack the formal details consistent with traditional proofs of secondary and post-secondary mathematics, it is a significant starting point on which educators and students can build to develop important proving and reasoning skills. In addition, we believe that valuing early student argumentation work can enhance student self-efficacy and empower students to value their own voice in an area of mathematics that is not often formally taught. Thus, given the value of argumentation at the elementary age, we sought to explore the use of mathematical argumentation with elementary school teachers.

Method

Using an action-research case study approach, we examined elementary teachers' perspectives on the implementation of argumentation in their mathematics programs. The case study approach enabled us to collect information relating to teachers' experiences and attitudes, as the main underpinning of qualitative research is that reality is constructed by individuals who interact with their social world (Merriam, 1998). Case studies allow the researcher to understand the case in all its complexity (Stake, 2000) and enable researchers to explore in depth an event, process, activity or one or more individuals, using a variety of data sources over a specific period of time to provide the fullest representations of inquiry (Creswell & Creswell, 2007; Stake, 1995). In this study, the case highlighted the journey of seven elementary teachers as they implemented argumentation tasks in their classrooms, using an action research approach.

Action research is a process where educators explore their own practice through meaningful inquiry in their classrooms (Katz & Stupel, 2016; McNiff, 2016; van Driel et al., 2014). It involves critical reflection on current practices and challenging current assumptions about the teaching/learning process (Carr & Kemmis, 1986; McNiff & Whitehead, 2011; Vaughan & Burnaford, 2016). It has its roots in educational reform where educators reflect on their own teaching (Cain & Harris, 2013; Elliott, 1991). By researching their own practice, teachers can gain insight and knowledge that can impact their own teaching and possibly their curriculum development (Glenn, 2017; Pella, 2015; Wells, 1994), resulting in a process that has the potential to empower educators. As Poetter (1997) describes, "The act of inquiry about practice and writing about it, and sharing results with others, gives voice for growth beyond the oftentimes private, isolated world of teaching practice" (p. 6). In addition, when teachers are given space to decide what needs changing and what will stay the same, they gain control over their own practice (McIntyre & O'Hair, 1996; McNiff, 2016) and inquiry becomes an integral part of their teaching (Cochran-Smith & Lytle, 2009; McNiff, 2016).

We specifically chose a collaborative action research model for this study as we felt it would provide us with insight into the teachers' actual experiences as they explored the implementation of argumentation in their mathematics programs. Collaborative action research invites teachers to explore an area of their teaching as a group. It brings teachers together to discuss and share ideas and thoughts about their educational research. As Manfra (2019) describes, "When working as part of a community of practice, action researchers engage in sustained professional learning activities" (p. 163). Professional learning is limited when educators do not have the opportunities to talk with each other about their practice (Darling-Hammond, 1998). McNiff and Whitehead (2010) stress that "[Action research] becomes a case of people working collaboratively to improve their practices by improving their learning about those practices, and checking with one another that what they know is valid" (p. 21).

The collaborative nature of action research can also provide a supportive context which can empower teachers to take risks and explore new teaching approaches (Aspland et al., 1996; van Dreil et al., 2014). Hence, in our view, if the purpose of action research is to reflect on personal experiences, then a collaborative approach to action research is paramount. Patterson and colleagues (1990) note that every teacher has stories to tell, and every teacher has truths to share. Action research can assist teachers in finding their own voices and sharing their own stories. Collaborative action research allows teachers to explore what is meaningful to them and in doing so, share their journey of exploration (McNiff & Whitehead, 2010).

There is no single overall effective model for conducting action research. What is critical is that the action research process includes elements of reflecting, planning, acting and evaluating (McNiff, 2010). This study integrates components from both practical and emancipatory action research as identified by Carr and Kemmis (1986). The direction of the research and the types of explorations that took place were the responsibility of the participants. The topic of argumentation was predetermined and we, as the facilitators, could be considered outsiders. These two characteristics are indicative of practical action research (Carr & Kemmis, 1986). Indeed, practical action research is an approach in which an outside facilitator identifies concerns and changes to attempt and assists with the effects of change. Facilitators also help the practitioners reflect on the effectiveness of the outcomes achieved through the research (Carr & Kemmis,1986).

Participants & Data Collection

This research was facilitated at two elementary schools in Southern Ontario, Canada from two different school districts. There were seven participants in total. Participants from school A included three teachers who taught grades 2, 2/3, and 4, respectively and a school librarian who taught a myriad of subjects to all classes. School B included three teachers who taught grades 2/3, 3, and 3/4, respectively. We have provided a summary in Table 1.

Table 1. Summary of Teacher Participants

Teacher	Grade	School
Brenda	School Librarian	A
Felicity	3	A
Kate	4	A
Susan	2/3	A
Danielle	2/3	B
Diana	3	B
Melissa	3/ 4	B

The study also included 131 elementary-aged students; 76 from school A and 55 students from school B. School A had a high percentage of English Language Learners (ELLs) and school B had a high percentage of students receiving Special Education Support. The recruitment of the participants took place via email. The researchers of the study sent out emails to various schools, asking for participants and chose the participants on a first come, first served basis. Data was collected from five sources: 1) Teacher research meetings; 2) teacher journals (which consisted of notes and observations from their lessons which they shared during the research meetings); 3) student work samples (argumentations); 4) student interviews, 5) videos of students 6) teacher interviews; 7) one demonstration lesson by author 1; and 8) one co-taught lesson with each teacher with author 1. The data shared in this paper is a subset of a larger study in which we explored the use of argumentation in elementary classrooms as a precursor to proofs; hence, our focus in this paper is primarily on data from sources 1, 2, and 6. To provide context for what teachers are sharing, we also share some student samples (source 3), but contend that our own analysis of source 3 is beyond the scope of this paper.

A demonstration lesson took place in the libraries of both schools A and B with all of the students and teachers participating in the research study from that specific school in attendance. The lesson began with Author 1 asking the students what they thought the word *proof* meant. After the students shared their ideas, Author 1 presented the statement 2+2=5 and asked if the statement was true or false. Once students confirmed that the statement was false, Author 1 asked students how they would prove that it was false. The students shared different strategies they would use such as visuals, numerical notations etc., to show it was false. Students were then invited to complete a similar argumentation task but in smaller groups and record their thinking. At the end of the lesson, students were offered the opportunity to share their argumentations with the larger group, after which the teachers and Author 1 met to debrief the lesson and student work. At the end of the lesson, Author 1 invited students who wished to do so, to share their argumentations with the larger group. Once the demonstration lesson was completed, the teachers and Author 1 met to debrief the lesson and student work.

Teachers at each school participated in a five-month action research project and met five times over the course of the study for 2-3 hours. The teachers explored the use of one-case argumentation tasks in elementary mathematics classrooms to determine its effectiveness in teaching argumentation in their mathematics program. The argumentation tasks involved providing students with number statements which they already knew to be false (e.g., 10 + 10 = 12; 6 × 4 = 25), and inviting them to argue, using visuals, numeric notation, and/or written explanations, why it was false. Because we felt that the goal of this approach was to serve as a precursor to formal proofs, we chose statements that students already knew to be false (based on consultation with their teachers) to allow students to focus on constructing an argument to convince others that the statement was false without having the burden of also trying to understand the mathematical concepts/ideas imbedded in the proof statement. We chose false statements, as one only needs a single contradiction to disprove a false statement, whereas to prove that a statement is true, one needs to show it to be true in all contexts for which a more complex argument needs to be constructed.

Over the duration of the study, the teachers explored different approaches to implementing argumentation tasks in their mathematics programs to assess their effectiveness. They were encouraged to video tape their students discussing ideas with each other, presenting their arguments to the class, and responding to teacher questions. Teachers also kept observation notes. During the research meetings, teachers compared student samples, and discussed their experiences of infusing argumentation tasks in their practice.

Approximately every three weeks the researchers met with the teachers. During these research meetings the teachers discussed how they explored the argumentation strategy in their practice. They shared student samples from their lessons and examined the various strategies (such as number lines, arrays etc.) that the students had used. The teachers also shared how they introduced the strategy and the kinds of questions they found helpful to push student thinking forward. They noted similarities and differences among the diverse student work samples and explored the challenges and benefits of the teaching strategy as it related to student learning. Some teachers shared videos of students completing and discussing their argumentations. The video recordings lead to teacher conversations about the kinds of student thinking and mathematical discourse the argumentation tasks fostered. These research meetings helped frame the next argumentation lesson as teachers would take specific ideas they had not tried, and/or improve an existing teaching approach in order to further their exploration on the impact of this teaching approach in their mathematics teaching.

During the last month of the study, Author 1 co-taught one lesson with each teacher. This lesson was co-planned in advance between the teacher and the researcher. During the lesson, the teacher and researcher gathered additional data through video recordings and field notes. At the end of the study, teachers were interviewed individually for approximately 45 minutes to an hour in order for the researchers to gain further insight on their perspectives of teaching argumentation in their mathematics program.

The Demonstration Lesson

Prior to starting to implement argumentation in their classrooms, teachers requested a demonstration lesson be done that they could observe. This lesson took place in the school library at each school with all of the student participants from that specific school and their teachers. During the lesson, the teachers circulated, and interacted with their students. While the students worked on their argumentations, the teachers asked the students questions and, in some cases, videotaped their students working and responding to questions. At the end of the lesson, the researcher asked the teachers to choose pairs of students to share their work with the whole group. During this sharing session, students described their argumentations and how they proved that the statement was false.

Data Analysis

Data was analyzed on an ongoing basis. With respect to data used in this paper, all interviews and research meetings were transcribed verbatim, with identifying names and locations being replaced with pseudonyms. Using the constant-comparison method (Elliott & Jordan, 2010; Glaser & Strause, 1967; Hubbard & Power, 2003), researchers cyclically coded data, beginning with a general coding cycle (guided by broad *a priori* codes generated from interview questions), followed by discussion, and then more focused coding centered on teacher perspectives on the implementation of argumentation. Authors remained in constant discussion about the analysis process to maintain consistency. Once data had been sufficiently coded, it was consolidated such that authors could analyzed the data for emerging patterns and themes (Saldaña, 2016). Though each of these processes were aided by the use of word processing software (i.e., by highlighting, underlining, and annotating transcripts, developing charts to consolidate codes and related data, etc.), communication between authors was key in developing common understandings of the data. The authors triangulated the data by comparing the themes and patterns gathered through the different data sources. The researchers analyzed the data sources individually and then met to communicate and compare their assessments. When discrepancies in analysis arose, the authors discussed and carefully re-examined the data sources to ensure consistency and to reconcile any differences. This included re-examining student samples for context and reviewing charted themes from the transcripts.

Results

Over the course of this study, teachers developed deeper understanding and greater confidence in using argumentation in their classrooms. Specifically, teachers' own learning about both argumentation as well as their students' capabilities evolved over time, the details of which will be delved into below.

Teacher Learning: Argumentation Demonstration Lesson

During our first research meeting, teachers provided context for their starting place in this study. The teachers shared that they did not have a deep understanding of argumentation and they did not explicitly teach argumentation skills in their practice. The teachers also noted that they had not previously engaged their students in formally proving or disproving mathematical ideas or statements.

Furthermore, they had not considered explicitly teaching mathematical argumentation skills through proofs nor inviting students to explore and discuss the kinds of mathematical arguments that they could use in their work. Thus, the teachers did not feel confident enough to implement an argumentation task like the ones implemented in this study. They expressed that they lacked knowledge of how to introduce the notion of argumentation, how to facilitate it, and most importantly, how to assess it. Further, the teachers were not familiar with the specific approach to argumentation that was used for this study, in which students were presented with a false statement and asked to disprove it, while working collaboratively in pairs.

To provide context for their usual teaching practices in mathematics, teachers shared that, prior to their participation in this study, they would typically invite students to justify their thinking as part of mathematical discussions. When students shared their answers, teachers would invite students to explain why a specific strategy worked or why an answer was correct. Several teachers also pointed out that they rarely invited their students to engage in group work as a regular part their practice. These teachers felt more comfortable with students working individually on more traditional kinds of activities using worksheets and textbooks. As Diana noted,

> I was concerned about the group work of [mathematical argumentation tasks with] this group, in particular—my class, in particular. There are a lot of hurdles with group work, [such as] accepting others' ideas, not wanting one person to do all of the work. Finding that balance [is tricky].

Because of their uncertainty around implementing argumentation in their classrooms and limited familiarity with associated pedagogies, teachers believed that a demonstration lesson provided by Author 1 would allow them to have a better understanding of what argumentation was and how it could be used to teach mathematical thinking. They also stressed that this would help them to gain insight into how their students would cope with such a task and how they worked collaboratively in pairs. Based on the teachers' request, author 1 co-planned a lesson with the teachers that Author 1 would teach to their students. Susan, a grade 2 teacher described,

> I was hesitant when I saw the demonstration lesson because I have a few students that I thought would not be able to work with a partner. So that concerned me, having to [have] them [work] together. But [the demonstration lesson] worked out better than I thought.

Danielle, a grade 4 teacher, elaborated on this, saying,

> Having [my students] work with a partner allowed them to talk [during the demonstration lesson]. They did not get stuck and maybe got ideas from their partners or saw what their partners were doing and even copied it. But at least they had done some strategies and next time were able to do the strategy on their own.

Before the demonstration lesson, the teachers also expressed concern that their students would not be capable of completing the argumentation tasks successfully. They felt that the mathematical argumentation task might be too challenging for those students who had historically struggled in math. The teachers were concerned that these students would become discouraged and shut down or that those who did not struggle would feel that the task was too easy. However, the demonstration lesson enabled them to see all of their students, regardless of their abilities, fully engage in disproving mathematical statements. Felicity, a grade 3 teacher describes her experience observing the lesson:

> [The argumentation task] was not as difficult as what I initially thought. Even the numbers that were chosen, like it could be something very simple, but yet their [student] thinking that goes into [it] is very complex. I thought, those are easy numbers! What will come of this? But I think it was intentional that the numbers are simpler so they [students] can get at the process of it. That is what I needed to see.

Observing the demonstration lesson proved to be pivotal in helping assuage some of the concerns teachers had about independently implementing mathematical argumentation in their classrooms. Importantly, as the teachers shared, this allowed them to feel more confident in attempting to integrate mathematical argumentation on their own.

Improving Access to Complex Mathematics: Engaging all Students

As teachers from both schools implemented the argumentation tasks in their practice, they found that the open-endedness and mathematical content of the argumentation tasks rendered them accessible for all students regardless of prior achievement level. They noted that because the mathematical statements involved mathematical content that their students already understood and had experienced in past grades, their students felt comfortable participating and had multiple entry points into the lesson. Melissa, a grade 3/4 (split) teacher, presented work samples from one of the lessons she co-taught with Author 1 in which the students were invited to argue whether the statement 4 + 4 = 9 was true or false. One student (Figure 1) drew a set of nine objects and highlighted two groups of four within the set of nine to show that two groups of four equals eight. They also drew nine facts and put an "X" under each one to show that 9 – 4 ≠ 8 and argued that "9 is bigger than 8 and we are taking away 4." Another student in her class (Figure 2), applied what he knew about fact families stating "8 – 4 = 9 is not true because 8 – 4 = 8 and 4 + 4 = 8." The student also used fractional concepts to argue that "4 is half of eight and 4 + 4 = 8 and so 8 – 4 is not nine." Melissa felt that the different approaches that were evident in the students' work demonstrated how the argumentation tasks not only accommodated all learners but offered opportunities for students to demonstrate their conceptual understandings in different ways. She also shared that another group of students looked at 8 as two groups of 4 and reasoned that, "If you take one group of 4 away, that would leave you with a group of 4." Each of these examples that Melissa shared served to underscore her point that students were able to demonstrate deep and varied understandings of number sense, even while starting from a seemingly simple mathematical statement. Indeed, students not only had diverse entry points, mathematically speaking, but they could also communicate these ideas in various ways using images, numerical notation, and/or words. Teachers believed that these were critical advantages for such an activity, and consequently, students who did not normally participate, engaged fully and frequently. Diana described the engagement opportunities the argumentation tasks afforded her students:

> I think [argumentation tasks] makes them think outside of the box…Everyone can jump in at their own level. Patti can even get in there. She really, really [was] struggling. She can even get in there. She can draw pictures. She can feel successful. And then all the way up to the Michaels of the world who find these really complex ways [of doing the task]. I think it is good when they show the other kids how they got it, too. I noticed that a lot of kids are always interested in how other kids solve things.

Teachers from both schools observed that by inviting their students to explore a false mathematical statement and prove why it was false, their students were able to explore specific mathematical concepts more deeply. At times, their students grappled with how to disprove something that they knew to be false. Teachers explained that this struggle enabled students to reflect on these math concepts and consolidate their understanding. Melissa documented students saying, "We are trying to use different tools from math that we have used, like number lines and fractions, but sometimes [the representation] does not work with this [specific proof]" Melissa discerned that students were evaluating their own mathematical ideas to determine which one would make a convincing argument. The teacher librarian, Brenda, described this process as students "drawing from everything they knew." Students' application of mathematical ideas allowed the teachers to build on their students' learning and move them forward in their thinking. The tasks did not only support learners who struggled. Sarah, a grade 2 teacher, noted that one of her students who was strong in mathematics, and who found the argumentation tasks easy explained, "[The argumentation tasks] helped me to stretch my thinking even more. Proofs are more like review to me. But it helps me to stretch my thinking." The teacher felt that this statement demonstrated that the argumentation tasks enabled all students to strengthen their problem solving and deepen their thinking regardless of their abilities.

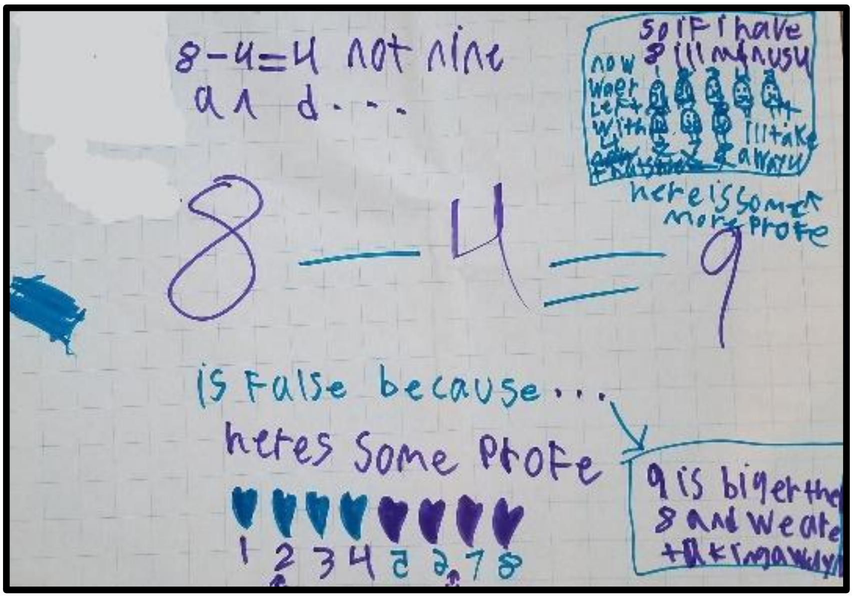

Figure 1. Student Argumentation Sample Disproving 8-4=9

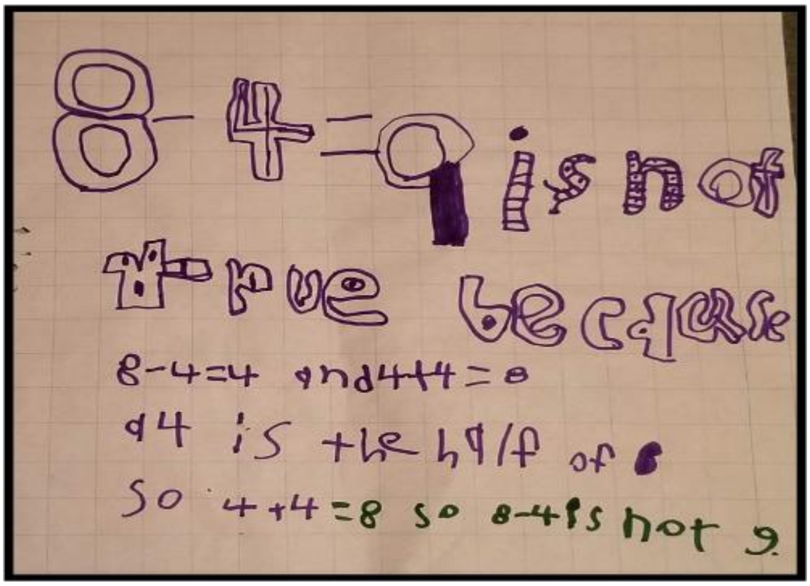

Figure 2. Student Argumentation Sample Disproving 8-4=9 using Fractions

Teachers from school A (where there was a high percentage of English Language Learners [ELLs]), were happy to see that the ELL students in their classes fully participated in the argumentation tasks. They explained that these students understood the concepts related to the mathematical statements and were able to demonstrate how they were false. Furthermore, the ELLs were able to represent their arguments through images and numbers (as well as words) which enabled them to communicate their mathematical ideas and reasoning very clearly for others to understand, regardless of their level of English. Felicity reflected that, “Our ELLs can participate and share very complex thinking as they demonstrate through images and numbers why a specific mathematical statement is not true.” While ELLs were not the focus of this study, Felicity’s observations, which were shared by all of the participants at school A, suggest that the argumentation tasks explored in this study hold promise for providing opportunities of engagement for ELLs in mathematics.

In addition to providing meaningful opportunities for students to engage in mathematical tasks, teachers also shared the impact the argumentation tasks had on the annual standardized test. The *Evaluation and Quality Assessment Office* (EQAO) assesses students’ literacy and numeracy skills each year on behalf of the Ministry of Education of Ontario. This test has a blend of multiple choice and open-response questions and is administered each year to students in grades 3, 6 and 9. While this standardized test was not intended to be part of this study, it was part of the grade 3 teachers’ and students’ experiences in their mathematics program during this study and thus provided another perspective related to the argumentation tasks. The grade 3 teachers from school B noted that historically, students who struggled in mathematics would rarely write anything down for the open-response problem-solving section of the EQAO test. However, the teachers perceived a definite change in student behavior the year that argumentation was implemented in their classroom as compared to years past. Diana explained:

> [On the EQAO test] this student using a number line was surprising as she would have never done something like this before. She would have given up. She would not have preserved or tried anything. And the fact that she was using different strategies [on the EQAO Test] was incredible. The only thing that has changed [this year] is doing the argumentation.

Diana also noticed that her students were more engaged with the open-response questions on the EQAO Test:

> For the open response [in the EQAO test], for sure [students] just do not write the answer any more, now they actually show how they did [it] and like, they point out how something is wrong. Like, it’s crazy...This group definitely does a better job of explaining thinking [than similar groups of students in the past]. For sure. They used pictures, they used words, they used, like, a variety of things to show how they got the answer, whereas before, they would have shown, like, rough work and that would have been it.

Without our prompting, the grade 3 teachers from school A shared that there was, what they felt was a significant gain in their students’ test results compared to the other grade 3 class in their school who were not part of this study.

Fostering Deep Mathematical Thinking and Connections

Teachers found that the implementation of argumentation tasks in their mathematics programs had unexpected benefits. In particular, they found that by using argumentation, students’ mathematical understandings were deepened, and they were able to make connections in numerous ways.

Application of Prior Knowledge and Transferability of Argumentation Tasks

The teachers from both schools observed that students infused mathematical ideas in their argumentation that they had not included in their lesson instruction. Teachers from both schools began to not only implement proofs as stand-alone tasks, but also used the thinking shared in student argumentations to support a teaching approach called number talks. Number talks are whole class discussions in which a teacher shares a number fact and students share the strategies they used to solve it. The teacher facilitates the discussion and notes the different strategies for all students to see (Parrish, 2010). The teachers felt that the different ways students could express their ideas in their argumentation tasks connected well to number talks. For example, when a group of students presented their argumentation that included a number line, a concept that their teacher, Sarah, had not yet introduced, the teacher revisited a number talk they had completed earlier that day and invited the students to explore connections between the number line and the concepts they explored during the number talk. Teachers shared that the argumentation tasks seemed to strengthen student engagement with the number talks. Diana described this change as follows:

> If you could compare the beginning of the year [with number talks] with what they are doing now, it's crazy. They are putting up decimal numbers, brackets. They are mixing multiplication, division. They love it. Even Josh will go up there. He struggles, yet [now] math seems to be the one and only thing that he feels the most [successful at]. He enjoys doing this [argumentation].

Teachers further argued that the mathematical ideas that students included in their argumentations readily supported number talks. Several teachers incorporated false mathematical statements as part of their number talks and invited students in a whole-class context to verbally explore why it was false. They found that this approach elicited a lot of rich mathematical discussions and seemed to strengthen students' argumentation skills as the teachers observed students applying some of the ideas shared during the number talks in their proofs.

The argumentation tasks became an integral part of the teachers' mathematics program. In addition to number talks, several teachers from school B extended the use~~d~~ of argumentation beyond number sense to probability where they wrote a false statement about a probability and invited their students to determine whether it was true or false. teachers from school B provided students with an image of a spinner where one student, had 2/3 of the spinner and the other student had 1/3 of the spinner. The students were provided the statement "This is a fair spinner," and were asked if the statement was true or false.

The teachers remarked that the tasks were easy to implement and that their students were able to readily transfer the argumentation skills they acquired from the number statements to disprove the false probability statement. Melissa explained:

> We could tie the proofs to the probability. It was easy enough to use in daily practice. It was not something that you had to reach for. It did not take a lot of time to prepare. It was there and accessible. It is user-friendly, teacher-friendly – something that you can turn to. It was not just on those days [when

> research meetings took place] and done with. It was embedded [throughout the year]. [The students] still talk about it. They say in their work, is this like we are doing the proofs?... It is nice to see that reflected in their mathematical thinking.

Other teachers corroborated Melissa's comments, suggesting that argumentation was readily applied to other areas of mathematics, in part because the teachers, themselves, found this way of engaging with mathematics accessible and reasonable to implement.

Student Collaboration and Learning

Teachers found that the argumentation tasks offered opportunities for students to learn from each other. Teachers from both schools concurred that inviting several students to share their argumentation in front of the whole class was extremely helpful as this enabled students to share and discuss their mathematical ideas in a whole-group context. In addition, students often took the ideas they learned from these presentations and applied them in their own work. For example, one pair of students showed how they used fact families in their argumentation while another pair, as mentioned earlier in this paper, used a number line. Kate explained that after that specific lesson, students who had not previously used number lines and/or fact families in their argumentations began to do so. This, she felt, provided evidence of students learning from each other.

At school A, the teachers who taught grade 3 often combined their classes to work on argumentation tasks. They found that by combining the classes they were able to observe each other's students and discuss their work. They noticed that students learned new ideas from each other, including those in a different class. Kate noted:

> I really did not work on number lines at all, and Felicity must have. Because by the time the third proof came around, all my kids were using number lines. I hardly use number lines. My children were learning from her children because that was something she taught. I know for sure that I did not teach it and they are all [using number lines]. They must have learned from each other.

The insights that the teachers shared throughout the study provided rich examples of how the argumentation task had an impact not only on their own practice, but also on how their students engaged with mathematics.

While the teachers did express concern about potential challenges early in the study, they did not speak extensively to challenges that arose during the implementation process. Among the early concerns teachers shared were that there was a limited amount of time the teachers had in their full math program to implement the argumentation tasks. However, they mitigated this by using the student argumentations as an onramp for teaching new concepts. They found that this approach was a practical use of time and student work. For example, as noted previously in this paper, Diana found that by using the student visuals and notations in the student argumentations as part of the number talks enabled her to save time and use the argumentations that students had already created for different learning contexts in her program. Similarly, other teachers found that the mathematical content shared in the student-created argumentations such as visuals, numerical notation and written descriptions, could be used to address possible misconceptions and/or review and underscore previous concepts that they had taught. Hence, the student argumentations became a time-saving teaching resource the teachers used for other lessons in their mathematics program.

Additionally, some teachers were concerned about students who struggled and how they would be able to explore argumentation and represent their thinking. Recognizing this, they decided to take special care to ensure that the false statements that they provided were understood by all students in their mathematics program. In doing so, the teachers found that students who struggled were able to fully engage in the argumentation task using ideas and notations that made sense to them. Teachers also noted that by providing opportunities, for those students who wished to do so, to share their argumentations with the class allowed their students to see different examples of mathematical thinking and representations. This sharing, the teachers felt, reinforced to their students that diverse sharing and representations of mathematical thinking were valued, creating a more inclusive learning context, especially for those students who lacked confidence.

Discussion

This study sought to investigate the ways that elementary teachers perceive the use of mathematical argumentation in their classrooms, including the impact that they saw. We found that while some teachers were initially hesitant to implement mathematical argumentation in their classrooms (Ayalon & Even, 2014; Bieda, 2010), when provided with support (such as with a demonstration lesson) they were able to confidently implement mathematical argumentation in their classrooms. Notably, even the teachers who were concerned that this way of doing mathematics would be challenging for struggling students (Makar et al., 2015; Stylianides et al., 2013) found it to have a positive impact in their classrooms.

By inviting students to disprove mathematical statements they already knew to be false, teachers noted that they were able to provide opportunities for students to deepen and extend their mathematical knowledge and reasoning skills (Brown, 2017; Staples & Newton, 2016). Furthermore, teachers found that this approach to argumentation resulted in students applying previously-learned concepts in creative, and reasonable mathematical ways to support their conjectures (Civil & Hunter, 2015; Rumsey & Langrall, 2016). In addition to applying previously learned knowledge, teachers also observed that students shared mathematical knowledge in their argumentations that they had not been previously taught (Rumsey, 2013) and employed argumentation strategies that they learned from their classmates, suggesting opportunities for collaborative learning afforded by engaging in mathematical argumentation hastened the development and sharing of mathematical knowledge.

The method of teaching argumentation presented in this study has implications for supporting and meeting the needs of diverse learners in mathematics. Several teachers noted that students who usually struggled and were hesitant to engage in mathematics activities, participated fully and successfully in the argumentation tasks. Working with mathematical content that they already understood seemed to encourage struggling learners to take more risks and challenge their thinking. The teachers felt that the openness of the tasks afforded entry points for all learners to create mathematical arguments, regardless of their achievement level (Rumsey, 2013).

At school A, which had high percentage of ELLs, teachers found that argumentation tasks were particularly well-suited for ELLs, as they could use a variety of images and symbols to communicate their ideas and share their thinking as they participated in group work. This kind of engagement is critical for ELLs in mathematics (Marks Krpan, 2014; Banes et al., 2018; Moschkovich, 2010). In addition, teachers shared that student argumentations provided a wealth of insights about their students' mathematical thinking, allowing teachers to explore these insights in depth through number talks and other mathematical lessons (Makar et al., 2015). Teachers were able to extend the mathematical argumentation framework outside of "simple" number sense questions and find rich ways to use it to deepen students' mathematical understanding across sub-disciplines (Schwarz et al., 2010; Staples & Newton, 2016; Whitenack & Yackel, 2002).

Prior to their engagement in this research, teachers were hesitant about implementing the argumentation tasks; however, once they became familiar with the approach, they not only infused the tasks in their practice, they also integrated them in other areas of their mathematics program outside the focus of this study. Teachers found the tasks accessible and relatively easy to infuse into their current math program and existing scheduling demands. Interestingly, even when provided the opportunity, teachers did not express major challenges during the implementation process. While we cannot speak to whether this means they did not encounter challenges at all, we feel it is important to highlight that teachers were instead focused on sharing their ideas to expand on argumentation tasks and continue doing this work in their classrooms.

Conclusion

The findings in this study have profound implications on not only for the value of argumentation in elementary mathematics classrooms, but on ways of engaging teachers in learning new and unfamiliar ways of doing and teaching mathematics. From a student perspective, we contend that the tasks explored in this study can provide access to argumentation for all students which can ease the cognitive burden of learning a new way of doing mathematics. Scaffolding argumentation in this way works to support students and teachers alike in developing confidence with argumentation itself. In addition, this approach can serve as an "on-ramp" for teachers who wish to infuse argumentation into their practice. In recognizing that argumentation matters for all students, we suggest that these kinds of tasks can play a role in supporting proof-building at all levels of learning, including secondary and post-secondary mathematics. Further, we argue that working with teachers through a collaborative, practical action research model where teachers are invited to experiment in a relatively low-stakes way with the support of other educators and researchers resulted in the empowerment of teachers to change classroom practices and dynamics and invite others to use similar models.

We believe this paper contributes to current research in mathematics education, providing an insightful examination of the implementation of mathematical argumentation in elementary classrooms, but recognize it is limited to the participants in this study: their unique experiences, and their specific school contexts. Therefore, while the findings may not be generalizable to every teacher in every school, this study does provide rich detail about the teachers, their thoughts, and experiences, and transferable ideas for other educators and researchers.

As we move towards greater access to data and a heightened need for critical, numerate thought, it is crucial that we work with students to develop an understanding of the (mathematical) world through means such as argumentation. In order to do so, it is imperative that the teaching of mathematics become more accessible for all educators. It is critical that we continue to empower and support teachers as they take risks in their practice to explore, reflect and engage in taking risks in their practice.

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