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Bima Sapkota
The University of Texas Rio Grande Valley, United States

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Abstract

Mathematics Preservice Teachers' (M-PSTs) conceptions of Mathematical Knowledge for Teaching (MKT) enhance their reflective skills because they utilize such conceptions to reflect on how to contextualize content knowledge during secondary mathematics teaching. While previous studies suggested M-PSTs develop MKT, including pedagogical content knowledge by analyzing teaching in video lessons, how M-PSTs enhance their conceptions of MKT through such analysis is underexplored. I used a collective case study approach to investigate how four secondary M-PSTs conceptualized MKT when they analyzed and discussed teaching represented in a video lesson using the MKT framework. The findings indicated that the M-PSTs often described teacher knowledge either as purely pedagogical or solely content knowledge in their initial analyses of the video lesson. After engaging in a discussion that introduced the MKT framework, the M-PSTs began to illustrate how a teacher's content knowledge influenced her instructional decisions and actions. Through this discussion, M-PSTs began identifying MKT as professional content knowledge unique to teaching and, thus, distinguished MKT from generic pedagogical or content knowledge. I argue that the language and concepts offered through the framework enhanced the M-PSTs' conceptions of MKT and suggest that the instructional activities from this study contribute to developing M-PSTs as reflective practitioners.

Introduction

Research has suggested that mathematics content courses do not adequately prepare mathematics preservice teachers (M-PSTs) to utilize their content knowledge for the work of secondary mathematics teaching (Ball & Bass, 2002; Wasserman et al., 2019). In addition, M-PSTs have limited opportunities to develop their competencies for analyzing mathematics teaching and learning through the lens of discipline-specific knowledge (Morris et al., 2009). In mathematics education, discipline-specific knowledge is often referred to as Mathematical Knowledge for Teaching ([MKT], Ball et al., 2008). MKT enhances teachers' instructional decision-making skills, ultimately increasing students' learning and achievement (Hill et al., 2005; Thomas et al., 2017). For example, teachers use their MKT to meaningfully select and sequence mathematical tasks when teaching particular mathematics topics (Sapkota & Huffman Hayes, 2024). Thus, mathematics methods courses need to include dispositions and language related to MKT as MKT assists M-PSTs in recognizing how content knowledge is applied in teaching (see Heid & Wilson, 2015). MKT includes two domains of teacher knowledge, subject matter knowledge (SMK) and pedagogical content knowledge (PCK). These domains are explained in the Conceptual Framework section below.

In recent years, M-PSTs' MKT and related practices have been developed through video analysis activities, such as engaging M-PSTs in attending to, interpreting, and responding students' thinking from a video lesson (Seidel et al., 2013). Several approaches in video analysis activities have been productive in developing M-PSTs' MKT. For example, providing M-PSTs with video lessons analysis framework (Santagata et al., 2007), reflecting individually on video lessons and sharing reflections with groups (Alsawaie & Alghazo, 2010), and watching and analyzing their own peer teaching video lessons (Nilsson, 2008) have been shown to have positive influences on developing M-PSTs' discipline-specific mathematical knowledge, including MKT. After analyzing teaching in video lessons based on given instructions such as highlighting and critiquing important events in classrooms, elementary and secondary preservice teachers (a) recognized significant aspects of classroom interactions and student learning (Alsawaie & Alghazo, 2010), (b) critically analyzed the effects of teachers' actions and instructional decisions on student learning (Santagata et al., 2007), and (c) acknowledged the importance of contextual factors (e.g., knowledge of students' social and cultural backgrounds) in teaching (Nilsson, 2008). In particular, these studies investigated how preservice teachers developed specific practices (e.g., teacher noticing). For example, Norton et al. (2011) investigated how preservice teachers learned to interpret and respond to students' thinking.

These studies also indicated that preservice teachers were provided with guided instructions (e.g., observation protocols, guided questions, or noticing protocols) while analyzing teacher and students' actions, including teachers' instructional decisions, students' problem-solving strategies, and/or students' mathematical thinking. The studies have demonstrated that analyzing their own teaching (e.g., teachers' instructional decisions and actions, students' actions and mathematical thinking) helps preservice teachers to grow as reflective practitioners (Nagro et., 2017). However, there are limited studies that discuss how utilizing language and disposition related to MKT in analyzing teaching in a video lesson would benefit secondary M-PSTs in becoming reflective practitioners. In particular, how the language and concepts offered through the MKT framework influence M-PSTs' analyses of teaching in general, including what teacher knowledge M-PSTs attend to and how they describe it, is underdeveloped.

In this study, I investigated how M-PSTs enhanced their conceptions of MKT as they analyzed teaching in a video lesson using the MKT framework. I argue that using the language and disposition related to MKT domains potentially enhance M-PSTs' conceptions of MKT and, hence, they would develop as reflective practitioners. As discussed previously, research has demonstrated that M-PSTs develop MKT by analyzing teaching. Through this study, I extended this research by investigating how analysis of teaching from a video lesson facilitate the development of M-PSTs' MKT.

Conceptual Framework: Mathematical Knowledge for Teaching

In this study, I use the conceptualization of MKT as defined by Ball et al. (2008). They defined MKT as, "content knowledge unique to teaching—a kind of subject-matter–specific professional knowledge" (p. 389). Teachers' MKT influences how effectively they present mathematical ideas to a class, how they respond to students' questions, and how effectively they build on students' mathematical ideas (Steele & Rogers, 2012). MKT extends beyond common content knowledge (CCK) and is broader than generic pedagogical knowledge. In addition to solving mathematical problems, teachers have to conceptualize how mathematical truths are established and from where the mathematical knowledge is originated. Emphasizing the content knowledge required for mathematics teaching, Ball and colleagues proposed a framework: "Domains of Mathematical Knowledge for Teaching" (2008, p. 403; see Figure 1).

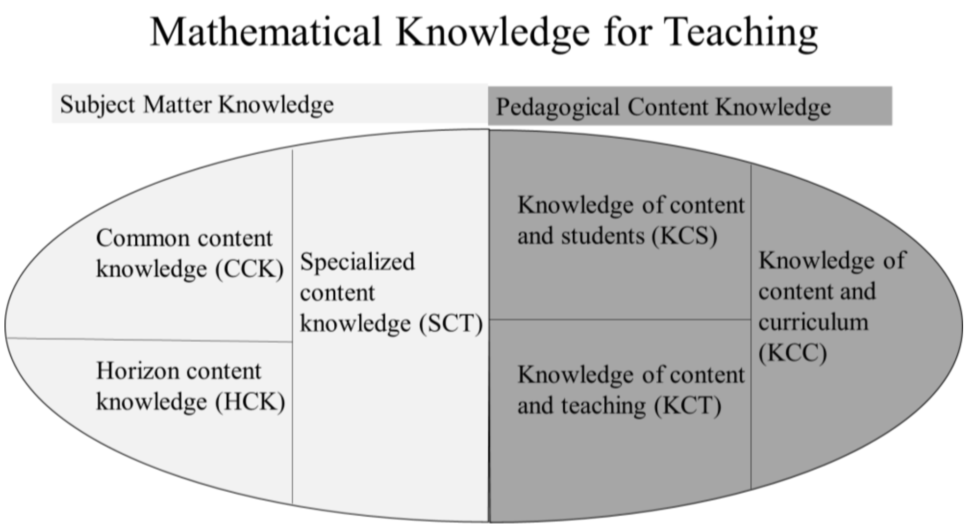

Figure 1. Mathematical Knowledge for Teaching by Ball et al. (2008, p. 403)

This framework has two domains of MKT, Subject Matter Knowledge (SMK) and Pedagogical Content Knowledge (PCK). SMK consists of three subdomains: (a) Common Content Knowledge (CCK), general mathematical knowledge not unique to the profession of teaching yet required to solve mathematical problems; (b) horizon content knowledge (HCK), knowledge of logical sequences of mathematical content; and (c) specialized content knowledge (SCK), mathematical knowledge unique to the profession of teaching (Ball et al., 2008; Nolan et al., 2015). SCK is distinguished from CCK in that teachers must conceptualize mathematical content in nuanced ways that include, but are not limited to, familiarity with several solution strategies of mathematical problems. The other domain, PCK, consists of: (a) knowledge of content and students (KCS), the understanding of students' mathematical knowledge, including their misconceptions and pre-knowledge (e.g., anticipating students' thinking and patterns of error); (b) knowledge of content and teaching (KCT), knowledge of effective instructional strategies required specifically for teaching mathematics (e.g., purposefully selecting mathematical examples based on students' prior knowledge); and (c) knowledge of content and curriculum (KCC), knowledge of learning outcomes and horizontal and vertical organizations of mathematical content over grade levels. Please see detailed descriptors of these subdomains in Sapkota and Huffman Hayes (2024). In this paper, I use the term "MKT domains" to refer to six subdomains.

Review of Relevant Literature

Mathematical Knowledge for Teaching in Practice

MKT emerged from the practice-based approach of teacher education; the subdomains of MKT were derived by observing elementary teachers' classroom practices (Ball et al., 2008). Wasserman and colleagues (2019) distinguished MKT from generic content knowledge and elaborated on how MKT draws from classroom practices; thus, it is related to a practice-based approach in teacher education. Holmes (2012) also defined MKT as professional knowledge that a classroom teacher needs to perform their teaching effectively, suggesting that MKT aligns with a practice-based approach. For example, teachers' HCK influences effective lesson planning because an understanding of big mathematical ideas helps teachers to make sense of students' responses and choose appropriate examples and non-examples (e.g., utilizing the concept of non-Euclidean Geometry to understand theorems from Euclidean Geometry; Wasserman & Stockton, 2013).

M-PSTs utilize the aspects of MKT in their analysis of teaching, and to develop their analytic quality (van den Kieboom, 2013). For example, M-PSTs utilize SCK to analyze how a teacher uses students' thinking through questioning. Similarly, M-PSTs' knowledge of content and students (KCS) indicate that M-PSTs should be able to formulate mathematical content and learning objectives in a way that students can make sense of MKT. In addition, Thomas et al. (2017) have argued that MKT domains are explicitly related to the practice of teaching. For example, a teacher's SCK assists them in interpreting students' thinking because they need to utilize the nuanced features of a child's particular ways of thinking and making sense of mathematical ideas. M-PST's MKT may affect their day-to-day classroom practices and their students' performance in mathematics in a long run (Nolan et al., 2015), and it also may influence their self-esteem as a mathematics teacher (Hill et al., 2005) and their self-efficacy in their professional practices (Alshehri & Youssef, 2022). Therefore, in this paper, I analyze how MKT is useful in analyzing teaching and, hence, developing as reflective practitioners.

Enhancing M-PSTs' MKT Through Video Analysis Activities

M-PSTs need to develop skills required for analyzing teacher knowledge and student thinking as this knowledge potentially helps them make instructional decisions in their future classrooms (Morris et al., 2009). Since teachers are expected to continue to improve their teaching (Conference Board of the Mathematical Sciences, 2012), analytic skills learned in teacher education programs may serve teachers throughout their career. M-PSTs' analytic skills related to teacher knowledge and student thinking allow them to make connections between theoretical and content knowledge learned from courses and classroom teaching experiences (Alsawaie & Alghazo, 2010; Özgün-Koca et al., 2020; Wasserman et al., 2019), which prepare them for teaching (Thomas et al., 2017). These analytic skills also help M-PSTs to develop MKT because MKT consists of teachers' knowledge of how to utilize students' thinking, students' prior knowledge, and the nature of mathematical knowledge to make instructional decisions (Ball et al., 2008). Analysis of teaching helps M-PSTs identify and analyze aspects of classroom events (e.g., classroom interactions, teacher questioning) in which instruction is successful in addition to other areas in which they need improvement (Alsawaie & Alghazo, 2010; Nilsson, 2008; Norton et al., 2011; Santagata et al., 2007). Therefore, providing M-PSTs with opportunities to analyze teaching in a video is a way to develop their MKT, and MKT assist M-PSTs in analyzing how to utilize their content and pedagogical knowledge in secondary mathematics teaching (Wasserman et al., 2019).

Santagata and colleagues (2007) highlighted how the use of video lessons in teacher education programs served as a means to develop elementary preservice teachers' abilities to make professional judgements. In their study, preservice teachers were provided with a video analysis framework to analyze learning goals, student thinking and learning, and alternative teaching strategies in video lessons. The findings indicated that the preservice teachers moved from describing what they observed in the classroom to analyzing the impacts of teacher actions on student learning as they progressed through video analysis activities. Their findings implied the need for guiding preservice teachers' analyses of teaching with specific instructions. Alsawaie and Alghazo (2010) also used questions related to student thinking, teacher actions, and classroom discourse to scaffold secondary M-PSTs' skills to notice and interpret noteworthy teacher and student actions from those questions. For instance:

> At about minute 28, a student asked the teacher: "What if we don't have a y-bisection?"
> What does this question tell you about this student's understanding of a y-bisection?
> What do you think of the way the teacher handled the student's question? (p. 229)

The preservice teachers shared their reflections via a digital discussion board and commented on each other's responses. Their findings indicated that throughout the activities, the preservice teachers' analyses of teaching shifted from describing classroom events in chronological style to highlighting noteworthy events in the classroom. The preservice teachers also began analyzing how and why classroom events were important for student learning. These findings demonstrated the significance of guiding preservice teachers' analysis of teaching through specific prompts to develop their PCK.

Similarly, McDuffie and colleagues (2014) provided elementary preservice teachers with four lenses (teaching, learning, power, and participation) and prompts for each lens (e.g., how the teacher elicited students' thinking) to analyze video lessons. The researchers intended to develop preservice teachers' noticing skills with a focus on equitable instructional practices. The findings indicated that preservice teachers began to analyze the influences of teachers' actions in students' learning after analyzing teaching, suggesting that the given lenses and prompts enriched preservice teachers' depth and foci of noticing. Norton and colleagues (2011) used M-PSTs' video-based predictions to assess preservice teachers' knowledge of student thinking. The authors intentionally posed tasks and follow-up questions to foster student thinking. Preservice teachers constructed and revised their models of student reasoning from videos wherein students were solving mathematical problems. Their findings demonstrated that M-PSTs developed richer models of student thinking when they watched a video at the end of the semester than the beginning of the semester.

The studies have indicated that explicit questions, frameworks, and lenses resulted in M-PSTs' in-depth and more complex analyses of classroom actions; M-PSTs began to attend to the influences of teachers' actions in students' learning. The focus of this study was to investigate how the language and dispositions related to MKT frameworks, as well as use of those disposition to discuss and reflect on video lesson, would enhance M-PSTs' skills to identify and analyze teacher knowledge. Since M-PSTs' analysis of teacher knowledge is related to their conceptions of MKT, this study contributes to understanding of how the language and framework related to MKT influenced M-PSTs' conceptions of MKT.

Overall, the studies have suggested when M-PSTs engaged in the analysis of teaching in video lessons, they began to attend to influences of teachers' actions in students' learning. However, in most of the studies, M-PSTs were prompted with the specific guidance related to those specific actions. In this study, I analyze how the language and disposition related to MKT assist M-PSTs in analyzing teaching in a video lesson and how M-PSTs' analysis of a video lesson changes as they progress through analyzing teaching in a video lesson. Since providing M-PSTs with appropriate language and disposition also facilitated them to grow as reflective practitioners, I argue that the instructional activities related to this study potentially facilitate the development of M-PSTs' reflective skills.

Current Study

I investigated how M-PSTs enhanced their conceptions of MKT as they analyzed teaching in a video lesson and engaged in discussion of the MKT framework. The following research question guided this study: how did M-PSTs describe MKT and generic Pedagogical Knowledge (PK) in their analyses of teaching in a video lesson? In this study, I used the MKT framework for two purposes. First, I provided the framework to the participants as a tool to interpret teacher and student actions in a video lesson. Second, I used the framework to analyze the data.

Even though the focus of this study was not to assess or develop M-PSTs' professional noticing skills, I used the concept of teacher noticing to understand how PSTs attended to classroom events, how they described those events as several bodies of teacher knowledge (e.g., PCK, SMK), and how they build on connections between bodies of teacher knowledge. Since the researchers have suggested the theoretical connections between the notion of teacher professional noticing and MKT domains, it is appropriate to use the definitions of teacher professional noticing to analyze how M-PSTs use the language and disposition related to MKT to analyze the MKT domains. Thomas and colleagues (2017) have suggested that MKT and the notion of teacher professional noticing share theoretical spaces. For example, a teacher's ability to attend to students' mathematical thinking is influenced by his/her knowledge of content and students (KCS), which is a dimension of the MKT framework (Ball et al., 2008, p. 403). Similarly, a teacher's skills to appropriately interpret students' thinking is connected to their SCK. Thomas and colleagues further mentioned that a teacher's MKT provides us with a framework to consider and analyze the knowledge that is required for effective mathematics teaching practices. Even though teachers' SCK is not sufficient for teachers' professional noticing, a teacher's SCK is crucial to be able to attend to and interpret students' mathematical thinking (Wasserman et al., 2019).

I adopted the conceptualization of noticing processes from Sherin and colleagues (2011) to define two noticing-related concepts: attend to and interpret. Sherin and colleagues defined two processes of noticing: "(a) attending to particular events in an instructional setting and (b) making sense of events in an instructional setting" (p. 5). I used the terms "attend to" and "describe," which correspond to the first and the second processes, respectively, when reporting my findings. As such, I investigated how M-PSTs demonstrated conceptualizations of MKT by analyzing how they attended to teacher and student actions in a video lesson and how they described the MKT domains and generic PK from those actions. Generic PK "comprises knowledge of teaching-learning situations that is applicable across different teaching subjects" (Harr et al., 2014, p. 1). In this study, the generic classroom events that could not be correlated with teacher knowledge specifically required for mathematics teaching — such as classroom management — were defined and coded as generic PK.

Methods

I adopted a collective case study approach (Stake, 2003; Yin, 2017). The purpose of a collective case study design is to explore and understand a phenomenon rather than assessing it (Arghode et al., 2017). The phenomenon of this study was the M-PSTs' conceptualizations of MKT and descriptions of MKT domains. Since the aim was to understand the conceptions of a group of individuals bounded within a certain context (Morita-Mullaney & Stallings, 2018), a collective case study was an appropriate design for the study. In this study, all the participants were secondary M-PSTs, who served as four cases, enrolled in the same course and who all received the same instruction. As such, a collective case study allowed me to draw generalized themes by exploring repeated themes across the cases (Baxter & Jack, 2008). In particular, a collective case study allowed me to compare across the cases to draw themes from the data in a way that the findings are replicable across cases. In a collective case study, it is also appropriate to report unique characteristics from each case. I compared across the four cases to draw generalized themes across the cases through reporting case-specific characteristics.

Participants

The study included four White male M-PSTs enrolled in a secondary mathematics methods course at a large Midwestern U.S. university. These participants are referred to by the pseudonyms Jeremy, Jonah, Mitchell, and Seth. The course was taught by a professor and a graduate student, with my involvement as an instructor for part of the semester. At the point of data collection, the participants were the only students enrolled in the particular course prior to their student teaching semester. They had previously completed some mathematics content courses but had not yet taken any mathematics pedagogy courses. However, they had completed two general education courses covering topics such as multicultural education and aspects of teaching as a career. Thus, this course provided these M-PSTs with their first exposure to mathematics education pedagogy.

Video Analysis Activities and the Data for the Study

I facilitated a series of instructional activities (i.e., video analysis activities) with the participants over three class periods. I conducted the instructional activities on a Thursday followed by a Tuesday and Thursday. Each session took approximately 45 minutes. During the instructional activities, the participants discussed, analyzed, and reflected on a video lesson both prior to and following an introduction to the MKT framework (Table 1). On Day 1, the M-PSTs watched first 20 minutes of a 50-minute video lesson from an eighth-grade classroom projected on a large screen. The video lesson contained the first lesson in a sequence of 12 lessons on exponential functions. During instruction in the video lesson, the teacher assigned students to derive exponential rules by drawing patterns from given exponential problems. Afterward, she asked them to perform similar mathematical problems using those exponential rules. The lesson involved a variety of mathematical activities, including repeating procedures to solve mathematical problems and identifying exponential rules based on given mathematical problems.

Table 1. Summary of Instructional Activities and Data

Day	Activities	Data
1	• M-PSTs watched a video of an eighth-grade lesson on exponential functions, individually wrote a short reflection paper, and shared their reflections with the class.	• Day 1 Reflection Papers • Transcription of Day 1 Discussion
2	• M-PSTs discussed the MKT framework • M-PSTs individually analyzed their Day 1 Reflection Papers using the MKT framework • M-PSTs discussed what they noticed about the domains of MKT in their reflection papers	• Day 2 Video Analysis Papers • Transcription of Day 2 Discussion
3	• M-PSTs watched the video again and analyzed the video using the MKT framework • M-PSTs discussed their analysis papers	• Day 3 Video Analysis Papers • Transcription of Day 3 Discussion

During the discussion, students engaged in both individual exploration and group discussions from a video, which was publicly accessible from the TIMSS website ("Timssvideo," 2013). I chose this video because it offered a variety of teacher and student actions, including reviewing students' prior knowledge, using manipulatives, and engaging students in discussions, which the M-PSTs could potentially observe and describe while analyzing teaching. Additionally, this activity included students' discussions and teachers' question-posing to small and large groups. Students also posed questions to each other throughout the discussions. The teacher asked questions related to mathematical procedures, recalling facts, manipulating symbols, as well as exploring patterns and significant mathematical ideas. For example, the teacher asked, "What's your base? Remember, your exponent has to be written to a base." In this question, the teacher prompted recalling the standard symbol of exponent expressions. The teacher further asked, "And in your head think if you can come up with the rule, after you've done these problems, for multiplying these exponents, see if you see a pattern developing on your own." Here, the teacher encouraged students to identify patterns. This variety of teacher and student actions had the potential to offer more opportunities for the M-PSTs to attend to various subdomains of MKT.

After watching a portion of the video lesson, the M-PSTs individually wrote short reflections (referred to as "Day 1 Reflections") based on the given prompts (see Appendix A). The prompts were generic as I did not aim to guide their analyses through the prompts. Then, the M-PSTs discussed their reflections with the class. For example, I prompted the M-PSTs to describe the actions of the teacher and students (e.g., asking a question, using manipulatives) that seemed important to them for teaching and learning mathematics. Instead of asking the M-PSTs to describe specific teacher actions (e.g., teachers' actions related to prompting questions), I asked them to describe teacher actions that they found important for teaching mathematics. These generic prompts allowed me to understand which teacher actions M-PSTs attended to and described and how the M-PSTs developed the language and dispositions related to MKT.

On Day 2, I introduced the MKT framework to the M-PSTs and facilitated a short discussion about the SMK and PCK domains. While introducing the framework, I described MKT as a professional content knowledge unique to teaching and briefly explained how MKT differs from generic PK. While describing MKT, I also explained definitions of each MKT domain with an example (Sapkota & Huffman Hayes, 2024). In addition, I provided each M-PST with the handout of the MKT-framework figure, a brief description of each domain, and an example of each domain. M-PSTs also engaged in a brief discussion about how and where the MKT framework could be utilized in teaching.

Afterward, the M-PSTs individually analyzed their Day 1 Reflection papers. I use the phrase "Day 2 Video Analysis Papers" to refer to these papers. In their analysis papers, M-PSTs were asked to identify and discuss the MKT domains they had mentioned in their Day 1 Reflections. In order to facilitate their analyses, I provided them with prompts (see Appendix A). For example, I prompted them to describe the teacher actions that were mentioned in their Day 1 Reflections that could be described as examples of MKT domains. In addition, I prompted them to describe the teacher actions that they did not mention in their Day 1 Reflections but could be described as examples of MKT domains.

After completion of this task, I facilitated a discussion (referred to as "Day 2 Discussion") with the M-PSTs about the video lesson and the MKT domains. As such, they discussed the MKT domains that they highlighted in their reflection papers and experienced in their university program courses based on the guiding questions (see Appendix A). On Day 3, the M-PSTs re-watched the portion of the video from Day 1 in pairs and explained the classroom events that were important to them for mathematics teaching and learning. The M-PSTs were given guiding questions (see Appendix A) before they watched the video. They were also instructed that they could pause the video to take note of pivotal moments. I also provided them with the video transcript and allowed them to use the transcript if they wanted to revisit classroom actions verbatim in their analyses. Following this task, the M-PSTs shared the classroom actions (referred to as "Day 3 Discussion") that they attended to from the video lesson and discussed those events in relation to the MKT framework. I collected the data throughout the activities. In particular, the data for the study included M-PSTs' video analysis artifacts and transcriptions of audio-recorded discussions collected on Day 1, Day 2, and Day 3 (see Table 1).

Data Analysis

The first round of data analysis involved bottom-up and top-down interactive modes of analysis (Chi, 1997; Sapkota, 2022), which included identifying codes and themes. Generating codes using a theory or framework is an example of a top-down approach, while refining codes based on the data and identifying new codes are examples of a bottom-up approach (Chi, 1997). I first coded the data using descriptive and simultaneous coding methods (Saldaña, 2016), a bottom-up approach. A descriptive coding method enhances a researcher's overall understanding of the data while simultaneous coding allows a researcher to apply "two or more codes to a single qualitative datum" (Saldaña, 2016, p. 266). As such, I used a descriptive coding method to understand the types of classroom events that the M-PSTs attended to in the video lesson.

Utilizing a simultaneous coding method, I identified M-PSTs' descriptions of multiple classroom events from the same instance. For example, while describing the classroom events in the video lesson, one M-PST responded: "I thought the teacher asked rapid-fire, surface level questions, and didn't wait for responses." I identified two codes from this response: "quality of teachers' questions," and "wait time for responses." Overall, I identified 19 codes (see Appendix B) relating to teacher actions (e.g., reviewing students' pre-knowledge), student actions (e.g., working with peers), student-teacher interactions, and other classroom events (e.g., seating arrangement). Afterward, I categorized the codes using the MKT domains (top-down approach). From this categorization, I identified themes that could be generalized across the cases. Those themes were: (a) explaining teacher and student actions either as purely pedagogical or solely content knowledge and (b) explaining common content knowledge as subject matter knowledge. In the second round of the analysis, I used a constant comparison method to refine the tentative themes and to identify unique case-specific characteristics under each theme. Even though a constant comparative method is primarily used to generate grounded theories, it is also appropriate to generate findings from any form of qualitative data (Merriam & Tisdell, 2016). I performed both within-case and across-case analysis. Initially, I analyzed the data by the data points; I first analyzed all participants' Day 1 Reflections then moved to the Transcription of the Day 1 Discussion and continued with the remaining data points. From this analysis, I refined the tentative themes. For example, I rephrased the first tentative theme, explaining teacher and student actions either as purely pedagogical or solely content knowledge, to incorporating content in pedagogical content knowledge. As such, when I analyzed the data the second time, I found that M-PSTs did not always describe examples as either pedagogical or content knowledge, but rather they began to incorporate content in their descriptions of PCK. Then, in the second round of analysis, on a case-by-case basis, I reviewed each participant's analysis of classroom events to track the shifts in their attention of classroom events before and after the MKT framework discussions. I looked for unique characteristics, if any, within each case from this analysis. Overall, from the second-round analysis, I drew generalized themes across the cases and identified examples within each theme.

During data analysis, I did not focus on the phrases that the M-PSTs used in their descriptions/analyses of teacher knowledge, rather, I examined evidence of these domains in their descriptions. Since the M-PSTs had not yet learned about MKT domains at the beginning of the data collection, I did not expect them to use the language from the MKT domains (e.g., SMK, CCK). To be consistent with the data collection, I also used the same approach (i.e., coding the data based on what language they used from the MKT framework). For example, one M-PST mentioned, "I thought it was very important that the teacher could introduce newer vocabulary and then describe the vocab in terms of older and more similar concepts." Even though the M-PST did not explicitly mention this as an example of SCK, I interpreted it as SCK because he described the teacher content knowledge as knowledge required for teaching mathematics. To validate the findings, I integrated two sets of data: the personal reflections of participants and recordings of focused-group discussions, as described earlier. Furthermore, incorporating the participants' voices in the Findings section served to strengthen the study's validity (Creswell & Miller, 2000).

Findings

I organized the findings into two themes identified from the data: incorporating content knowledge in PCK and describing common content knowledge as SMK. Under each of these themes, I explained (a) how each M-PST identified and analyzed the MKT framework domains and generic PK, (b) how such recognition and analysis changed, or did not change, before and after the introduction to the MKT framework, and (c) the similar and unique characteristics across and within the cases in terms of M-PSTs' conceptions of MKT.

Distinguishing Generic Pedagogical Knowledge and Pedagogical Content Knowledge

M-PSTs attended to the teacher's questioning strategies (e.g., types of questions, wait time for responses), classroom management (e.g., seating arrangement), and student engagement in individual and group work (e.g., group discussion) in their Day 1 Reflections. They described these events as KCT and KCS in their Day 2 Analysis, suggesting that M-PSTs often considered PK as generic PCK in their Day 1 Reflections and Day 2 Analysis. M-PSTs identified the role of teachers' generic PK in effective teaching and learning; however, they did not seem to identify PCK as knowledge required for mathematics teaching in their Day 1 Reflections and Day 2 Analysis. As such, they often excluded content while describing classroom events related to PCK (Figure 2). They seemed to focus on one aspect of teacher knowledge either as content only or pedagogy. They could not explicate relational thinking while observing the video lesson to integrate both content and pedagogy in the first observation. However, they seemed to transition from distinctive categorical thinking of teacher knowledge as content focused or pedagogy focused to more integral aspect of PCK in the subsequent observations of the video lesson.

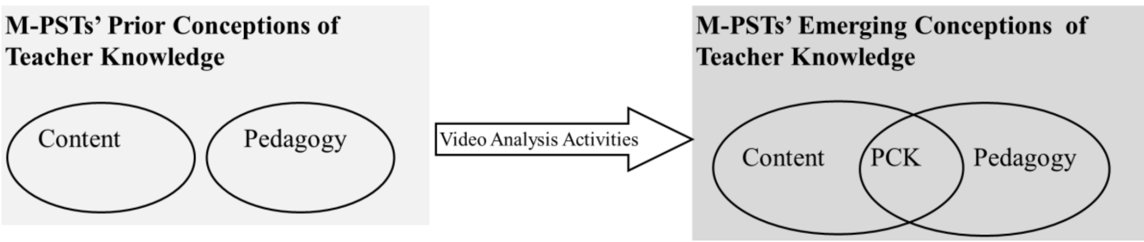

Figure 2. M-PSTs' Prior and Emerging Conceptualizations of PCK

In their Day 3 Analysis, M-PSTs continued to attend to the teacher's questioning strategies and student engagement, while also appreciating how the teacher's content knowledge was embedded in her instructional decisions related to teacher questioning and student engagement. I discuss, case by case, the details of this theme in the following paragraphs.

The Case of Seth

A participant, Seth, attended to the classroom events that included teacher questioning and student engagement in his Day 1 Reflection and Day 2 Analysis of the video lesson. His initial descriptions of KCT and KCS often did not include the teacher's knowledge of exponential functions; he appeared to have thought that PCK was the same as generic PK. In his Day 2 Analysis, Seth mentioned "asking too many questions too quickly, and group work after individual exploration pertain to both KCS and KCT because both are concerned about students and their predicted thought processes along with instructional techniques that come from knowledge about teaching." This response suggested that he seemed to think that the teacher's anticipation of students' thinking could be interpreted as KCS, while different teaching techniques (e.g., group work, individual exploration) were examples of KCT. He seemed attending to 'questioning' as a key feature in the lesson. While he seemed to have paid attention to "students' thought processes," he did not mention how the teacher's in-depth knowledge of the exponential function (e.g., various representations of exponents, pre-knowledge required to learn exponential functions, potential student misconceptions) was required to anticipate students' thinking in that particular context. During the Day 2 Discussion, he noted that he could only find examples of PCK (i.e., KCT and KCS) in his Day 1 Reflection because "[he] was not really focusing on what the teacher was saying but more about what she was doing and how she was saying things." His reflection demonstrated less of semantic and more of syntactic interpretation of the classroom process in the video lesson. This comment indicated that he seemed to interpret PK, what the teacher was doing, and content knowledge, what the teacher was saying, as two separate bodies of teacher knowledge. Thus, Seth did not seem to yet recognize that teachers' content knowledge constitutes parts of their PCK.

The Case of Mitchell

Another participant, Mitchell, attended to classroom events that included students' engagement in mathematical activities. In his Day 1 Reflections and Day 2 Analysis, Mitchell attended to some events related to PCK domains, but he did not mention how the teacher's in-depth knowledge of exponential functions influenced her teaching strategies. He mentioned that "hunting for patterns seems to really engage students." While he acknowledged that students were engaged in a meaningful mathematical activity, Mitchell did not acknowledge that the teacher utilized her knowledge of exponential rules to engage students in the pattern-finding activity. He seemed to reflect on the nature of the task or activity that engaged students. His notion of 'hunting for patterns' demonstrated his awareness of students' focus on the structure of mathematical tasks (possibly with data) during the engagement. During the Day 2 Discussion, he mentioned, "KCS and KCT were the two domains I was thinking that I am goanna look for; how well the teacher engaged the students, how well she knew the students, how well she is actually teaching the content." This reflection demonstrates his notion of 'engagement' and 'knowledge of students' as key aspects in the teaching. In addition, he mentioned, "I saw multiple examples of KCS and KCT in my reflections over the video. This includes how the students ask questions, their engagement, visual aids, and repetition." These comments suggested that Mitchell assumed how well the teacher presented the content, how she engaged students in classroom activities, and how well she knew the students' pre-knowledge, were comprised of KCT and KCS. As such, Mitchell considered the teacher's knowledge of her students and her classroom actions related to student engagement to be PCK. Yet, Mitchell did not mention how the teacher's content knowledge would be essential to productively utilize students' prior knowledge during mathematics teaching.

In his Day 3 Analysis, in addition to mentioning the teacher's questioning as a part of KCT, Mitchell acknowledged that the teacher's knowledge of exponential functions made mathematics teaching effective. This finding suggested that he had begun to recognize the role of the teacher's content knowledge in her instructional decisions in his Day 3 Analysis. He stated, "I thought it was very important that the teacher could introduce newer vocabulary and then describe the vocab in terms of older and more similar concepts." He seemed to acknowledge that the teacher's knowledge of vocabulary related to exponential functions made her teaching effective. In his explanation of KCS, Mitchell acknowledged that the teacher's KCS influenced her directions to her students as she had to understand their pre-knowledge. In his Day 3 Analysis, he mentioned that the teacher having knowledge of various representations of exponential functions was an example of KCT. He gave the following example of KCT: "Using the blocks as a visual representation and the graph as well is a good tactic." This comment suggested that Mitchell appreciated, to some extent, the role of the teacher's content knowledge in her selections of instructional strategies. During the Day 3 Analysis, Mitchell also mentioned that "KCS and KCT seemed like to be the most important things to focus on in the video, it seemed like we should have been focusing on the stuff she was specifically doing like how she's using blocks to show exponential functions." Mitchell's comments suggested an assumption that the teacher's KCS and KCT corresponded with student engagement in classroom activities. Mitchell also identified the teacher's use of visual aids as an example of PCK, suggesting he considered the teacher's content knowledge as part of PCK. However, Mitchell continued to consider generic PK as examples of KCS in his Day 3 Analysis. He described how the teacher directed her students in the classroom as an example of KCS. He mentioned that "the teacher seems to be teaching at students and is very strict in almost controlling how they do things instead of letting the students think freely and teaching/learning with students."

The Case of Johan

The Day 1 Reflection and Day 2 Analysis of another participant, Johan, suggested that he attended to classroom events related to PCK domains, but he also often described generic PK as PCK domains (i.e., KCS and KCT). The generic PK appeared as classroom discussions, student engagement, and the teacher's questioning during the lesson. In his Day 2 Analysis, Johan recognized that the teacher knowing her students' pre-knowledge was an example of KCS: "The teacher had to know where the students were in their understanding of materials; she had to know how much information to give students." Johan seemed to acknowledge that the teacher had to know her students' prior content knowledge to present new content effectively in the classroom. Yet, he did not explain how the teacher's knowledge of exponential functions (e.g., definition of exponential functions, representations of exponential growth) influenced the way she presented the content in the classroom. This suggested that he attended to KCT and KCS, while not mentioning the role of a teacher's content knowledge in her instructional decisions.

Johan often seemed to exclude the teacher's knowledge of exponential functions when explaining KCS and KCT in his Day 3 Analysis. He commented, "by having students wait to turn over papers, then work individually, then as a group, the teacher demonstrated knowledge of content and teaching." Giving an example of KCS, he stated that "the teacher had to assume the students know certain concepts at the beginning and the teacher knew the names of students." These examples indicated that Johan described generic events, which can happen in any classroom regardless of discipline, as examples of PCK domains. He did not describe how the teacher's knowledge of exponential functions and knowledge of patterns of students' errors influenced her instructional strategies.

However, during the Day 3 Discussion, Johan described the way that the teacher used various representations of exponential functions as an example of KCT: "The teacher knowing various representations is knowledge of content and teaching as it would help to have some other representations so students can make connections." This example indicated that Johan began to recognize KCS and KCT as teachers' professional knowledge required for teaching mathematics. He seemed to reflect on teacher action, student engagement and teacher knowledge of the content and students. This demonstrated that Johan was able to move from descriptive reflection on the teaching video toward more interpretive reflection with his noticing of interrelation of content knowledge to pedagogical practices.

The Case of Jeremy

Jeremy often offered a generic explanation of KCS and KCT in his initial reflection and analysis of the video lesson throughout the video analysis activities. He often described the events wherein the teacher engaged students in group discussions as examples of KCS and KCT, noting that "the teacher went around and asked a lot of questions." His noticing about 'walk around' activity demonstrates his understanding of teacher role in continuous formative assessment as a part of pedagogical process that integrates 'questioning' as an approach. Jeremy also mentioned that the teacher "making students work together and break[ing] questions down" provided examples of KCS and KCT. This reflection showed Jeremy's noticing of social aspect of classroom dynamics as a generic or common activity in the video lesson. In these comments, one event, breaking down questions, was specific to mathematics teaching and learning, while the other two events were not specific to a mathematics classroom; these events consisting of asking a lot of questions and engaging students in group work could occur in any classrooms regardless of discipline. Jeremy's responses suggested he attended to some specific examples of PCK, such as breaking down questions, in his Day 1 Reflection of the video lesson, while he also seemed to assume generic PK as PCK (i.e., KCS and KCT). His explanation about breaking down questions indicated that he might not have recognized the requisite in-depth knowledge of exponential functions that a teacher needs in order to ask follow-up questions even though he described content knowledge in PCK. During the Day 2 Discussion, Jeremy explained that he "mostly focused on KCS and KCT in [his] Day 1 Reflections as [he] talked about how the teacher was using the blocks to teach the exponential function." In this comment, Jeremy seemed to recognize that the classroom events related to the teacher's use of instructional materials were examples of PCK. However, he did not note that the teacher had to have knowledge of various representations of exponential functions, the mathematical content, to present them effectively to the class. This finding indicated that he attended to teacher actions that consisted of PCK domains, while he did not seem to identify them as examples of PCK.

Jeremy, in his Day 3 Analysis, continued to focus on how the teacher used several directions and questions to engage students in group work and individual exploration. Unlike in his Day 1 Reflection and Day 2 Analysis, Jeremy incorporated content in his descriptions of PCK (KCT in particular). For instance, during the Day 3 Discussion, Jeremy stated that the teacher's directions were ambiguous: "[The teacher] says if you need to expand it out to get the answer like I did that's what I would like you to do, which is a little ambiguous. This event seems to be pretty much aligned with knowledge of content and teaching." This response suggested that Jeremy discussed KCT specifically in the context of mathematics teaching and learning (i.e., the teacher's direction about exponential functions) even though he did not further build on how the teacher's content knowledge was associated with "ambiguous directions", despite his awareness of 'uncertainty' and 'complexity' of problem solving in exponential functions.

Overall, in their Day 1 Reflection and Day 2 Analysis of the video lesson, the M-PSTs often attended to generic classroom events, including seating arrangement, student engagement, teacher questioning, and individual and group work. The M-PSTs described these events as PCK. In their Day 3 Analyses and Discussions of the video lesson, M-PSTs began to describe how the teacher's content knowledge was related to her instructional decisions, suggesting that they had begun to incorporate both content and pedagogy in their descriptions of PCK. These case analyses demonstrated how M-PSTs might develop their noticing skills through repeated observations and iterative reflective thinking in the context of their attending to the key events in the teaching video.

Describing Subject Matter Knowledge (SMK) as Common Content Knowledge (CCK)

In their Day 1 Reflection and Day 2 Analysis of the video lesson, M-PSTs attended to several SMK-related events. They tended to describe SMK in a way that generalized it as CCK (Figure 3).

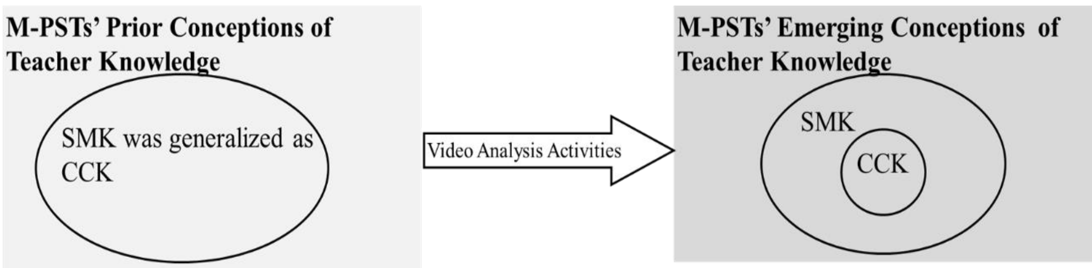

Figure 3. M-PSTs' Prior and Emerging Conceptualizations of SMK.

While M-PSTs recognized that teachers need to have knowledge of mathematical content, they did not seem to distinguish between the common content knowledge (e.g., solving mathematical problems) and the professional content knowledge (i.e., SCK and HCK) required for effective mathematics teaching. This could be because they were not explicitly exposed to the framework of mathematical knowledge for teaching at the beginning of video lesson observation. In the Day 3 Analysis, M-PSTs acknowledged, at least to some extent, that the teacher's mathematical knowledge influenced her teaching strategies, including questioning and directions. In the following sub-sections, I provide detailed descriptions of each participant's attention to and descriptions of the classroom events relating to SMK.

The Case of Seth

During Day 2 Discussion, Seth mentioned that he did not discuss SMK in his Day 1 Reflection of the video lesson. He responded, "I was not really focused on what the teacher was saying but more about what she was doing and how she was saying things more, like, more focused on the interactions than what she was saying." From this response, he seemed to assume that what the teacher presented in the classroom (i.e., what the teacher was saying) was a part of SMK, but how she facilitated classroom activities (i.e., how the teacher was presenting the content) was not a part of SMK. In the same discussion, giving an example of his university mathematics professor, Seth said, "the professor would literally just read straight from the book. He would read and do nothing but the book." Seth thought that his professor "lacked the pedagogical content knowledge" because "the professor knew the material, he could do examples, understood the material, but he just read it from the book and did not teach it." This comment suggested that Seth seemed to assume the professor "know[ing] the material" was an example of CCK. While he identified that the professor could not find alternative examples to represent the material, he did not identify how this example could represent the lack of his professor's professional content knowledge because he explained this event as an example of pedagogical knowledge.

In his Day 3 Analysis, Seth described the event where the teacher demonstrated various representations of exponential functions as professional content knowledge (i.e., SCK and HCK). During the Day 3 Discussion, Seth provided an example of teacher knowledge that is specifically required for mathematics teaching commenting, "she showed visually with the blocks, she showed the exponents graphically and numerically. She showed the mastery of different ways that you can represent the mathematics." In this response, he acknowledged that the teacher knowing different representations of content made mathematics teaching and learning effective, suggesting that he had begun to attend to and describe teacher professional content knowledge in his Day 3 Analysis. For him, teacher's knowledge and practice different representations of exponential functions could be considered as CCK.

The Case of Mitchell

Mitchell also tended to describe SMK as CCK in his Day 2 Analysis and Discussion. During the Day 2 Discussion, Mitchell mentioned that he did not discuss SMK in his Day 1 Reflection because he thought a teacher knowing the content was his basic expectation: "In my reflection, I almost had no, nothing from the subject matter knowledge, I was just thinking I am gonna look for how well the teacher engaged the students, how well she knew the students, how well she is actually teaching the content." This response indicated that he seemed to assume how well the teacher presented the content in the classroom had a limited connection to her content knowledge, indicating that he did not seem to identify SCK and HCK. In his Day 3 Analysis, however, Mitchell identified the relationship between the teacher's content knowledge and her choices of instructional strategies. He explained, "I thought beyond common content knowledge now, and [the teacher] showed a fairly effective teaching ways when she used new concepts and new math words, she broke them down in a way student know them." This reflection demonstrated that Mitchel was more aware of teachers CCK after being familiar to the MKT framework than without it at the beginning. In addition, he asserted that "evidence of a student trying to connect the graph of 2^x to a parabola was great, but the teacher did not build on this." Mitchell described this event as HCK, suggesting he acknowledged the significance of the teachers' understanding of a logical sequence of content in mathematics teaching and learning.

The Case of Johan

Johan, in his Day 2 Analysis of the video lesson, stated that he did not discuss SMK in his Day 1 Reflection. Similar to other M-PSTs' responses, Johan explained that he assumed what a teacher has to know as an expectation in his Day 2 Discussion. Yet, in his Day 3 Analysis, he described the classroom events related to SCK. He stated, "It felt like [the teacher] should know material in general. Now, I think back and feel that the teacher was talking about other ways that we can connect to other areas [SMK domains] …I think she knew *various ways that would connect to different areas* [emphasis added]." In his Day 3 Analysis, Johan stated that the teacher demonstrating the graphical and visual representations of exponential functions was an example of SCK as it showed her in-depth knowledge of the content. With more exposure to the MKT framework, he was able to notice the important events not only in teacher actions, but also in teacher's alternative approaches to present the material to the class, such as visuals and graphics. He said, "I think it showed her in-depth knowledge of exponents, it would help to have some other representations so students can make connections." This example indicated that Johan admitted the teacher's knowledge of various representation of the content influenced her instruction. He seemed to notice that the more the teacher had deeper sense of SMK, the more she was able to use different representations flexible ways.

The Case of Jeremy

Also, Jeremy often tended to describe SMK as CCK. In his Day 2 Analysis, he stated that he identified SMK in the video lesson as he noticed that the teacher understood exponential growth, indicating that he assumed SMK was limited to the teacher's understanding of the content. During the Day 2 Discussion, he mentioned that "students were figuring out those specific equations, like, exponential functions, and this is an example of SMK." In the Day 3 Analysis, Jeremy stated, "the teacher lays some groundwork about exponents and exponential growth." Even though he did not mention that this was an example of SMK, he seemed to acknowledge that a teacher having a basic knowledge of exponential functions was necessary for effective mathematics teaching. After multiple observations and deeper analysis of the video lesson, Jeremy seemed to reflect deeply on the teacher's SMK as CCK with a connection to the foundations of exponential functions as a prerequisite for effective teaching.

In summation, in the beginning of the activities, the M-PSTs tended to describe SMK as CCK, suggesting that they did not acknowledge SMK as professional content knowledge required for mathematics teaching. After analyzing and discussing the video lesson using the MKT framework, they began acknowledging that SMK, including SCK and HCK, was broader than CCK, and they identified that SMK influenced the teacher's instructional decisions. However, the M-PSTs did not yet fully conceptualize how SCK and HCK were related to effective mathematics teaching. The results based on M-PSTs' reflections on video lessons may provide an initial conceptualization of their MKT that requires a prolonged reflective thinking throughout the courses in a preservice teacher education.

Discussion

The key findings in this study suggested that, in their initial analysis of the video lesson, M-PSTs often categorized classroom events into "content" and "pedagogy," resulting in their evaluation of teacher knowledge either as purely pedagogical or solely content knowledge. As such, M-PSTs divided classroom events into two categories: "what teachers do" vs. "how teachers do," referring to SMK and PCK respectively. They did not establish a connection between these two categories. This finding supports the notion that PSTs' conceptions related to effective mathematics teaching and learning are still developing when they come into teacher education programs (Sapkota & Amanda Hayes, 2024; Thomas et al., 2017). Thus, teacher education programs should provide PSTs with opportunities to enhance those conceptions. Even though Thomas et al.'s (2017) claim was for elementary PSTs, my findings indicated that secondary M-PSTs may also have underdeveloped conceptions about mathematics teaching and learning until they receive opportunities to learn about teacher knowledge.

M-PSTs' descriptions of teacher knowledge either as purely pedagogical or solely content knowledge could potentially be due to limited opportunities that they have in teacher education programs. In one hand, the mathematics courses mostly might be taught by mathematicians without consideration of pedagogical aspects, and on the other hand the pedagogical courses might have been taught by educators without linking them in mathematical contexts. In particular, the M-PSTs in my study had taken mathematics content courses and generic education courses at the point of the data collection. In the generic education courses, they often learn about generic PK, which is different from PCK. Therefore, my finding supports the argument that undergraduate-level mathematics content courses do not prepare M-PSTs for making connections between content knowledge and secondary mathematics teaching (Ball & Bass, 2002; Wasserman et al., 2019).

After analyzing a video lesson using the MKT framework, the M-PSTs, to some extent, began recognizing the role of SMK in teachers' choices of instructional strategies. The M-PSTs admitted that the brief discussion on the MKT framework helped them to think about roles of the teacher's knowledge of exponential functions in her instructional choices. This finding suggested that providing M-PSTs with professional language (i.e., MKT framework) is productive to enhance their conceptions of teacher knowledge. While M-PSTs were not required to use the framework language in their initial analyses of the video lesson, and nor did I look for the framework language in their descriptions, M-PSTs' descriptions of teacher knowledge suggested that the language and concept offered through the framework enriched their understanding of teacher knowledge. Their reflective ability had changed from descriptive reflections to more interpretive reflections while going from Day 1 to Day 3 observations and reflections. The more they were exposed to the language of PCK, the more they were able to notice the important events in the lesson and making sense of them within the framework of MKT. These findings are similar to the findings of my previous study (Sapkota, 2022). In that study, I found that after reading and discussing two frameworks related to mathematical tasks, M-PSTs considered a broad set of factors, including student-related factors (e.g., student prior knowledge) and contextual factors (e.g., class time constraints) in their task descriptions. The use of formal language from the task frameworks seemed to have allowed them to produce a broad set of task descriptors and to provide more nuanced explanations for their task-related decisions. Therefore, the findings from both of these studies indicate that the instructional activities from this study are beneficial in terms of developing PSTs' reflective practices proposed by AMTE (2017).

Besides the use of the framework, the findings of the study could also potentially be influenced by M-PSTs' engagement in analyses of teaching in video lessons. The influences of analyzing, discussing, and reflecting on video lessons in PSTs' PCK development were also discussed by Santagata et al. (2007), Alsawaie and Alghazo (2010), and McDuffie et al. (2014). Santagata et al.'s (2007) findings indicated that PSTs' attention moved from superficial observations of teachers' and students' actions to evaluation of the effects of teachers' actions on student learning after analyzing video lessons using guided instructions. Alsawaie and Alghazo (2010) found that M-PSTs learned to pay attention to student learning and their thinking after analyzing and discussing video lessons based on guiding questions. McDuffie et al. (2014) aimed to develop PSTs' noticing abilities through video analysis activities and purposefully provided specific prompts to their PSTs. Since PSTs utilized specific frameworks or instructions to analyze teaching in these three studies, such changes seem more obvious than this study's findings. Therefore, the findings from this study demonstrated that M-PSTs, even without guided prompts or questions, can enhance their conceptions of MKT if they are aware of different sub-domains of MKT and their relationship with CK and PK as parts of PCK. In addition, the video analysis activities in previous studies were used to develop PSTs' knowledge/practice related to specific MKT domains (e.g., anticipating students' thinking). The findings from this study indicated that we could also use video analysis activities broadly without focusing on some domains to enhance M-PSTs' MKT. When M-PSTs are engaged in reflective practices in the context of observations of video lessons of teaching mathematics, they are more likely to be aware of important mathematical subject matters (CK), activities and tasks to engage students (PK), and connecting the subject matters with the contexts (PCK) to build their KCT and KCS (Nolan et al., 2015).

The M-PSTs still tended to interpret content and pedagogy as distinct bodies of teacher knowledge, rather than as interconnected bodies of knowledge, even after participating in the discussion that introduced the MKT framework. Because of the time constraints, the participants engaged in two short discussion sessions about the MKT framework. Given the complexity of the framework, these short discussion sessions were not enough to reasonably conceptualize the MKT framework domains. If the M-PSTs engaged in more discussion and reflection sessions similar to those in this study, they likely would have developed a more concrete understanding of MKT. Therefore, the findings suggested that developing PSTs' conceptions of MKT is a challenge for mathematics teacher educators because it does not happen instantly. Furthermore, the limitation of these findings lies in the small sample size of only four participants who share similar backgrounds. A more diverse participant pool would likely yield different results. Another limitation is related to my knowledge about MKT subdomains. I might not have identified some subdomains from the data due to my limited knowledge.

Conclusion and Recommendations

M-PSTs potentially conceptualize that teacher need specific professional content knowledge for effective mathematics teaching when they engage in analysis of teaching and discussion on the MKT framework. When M-PSTs attend to teachers' and students' actions and describe those events using professional language (e.g., MKT framework), they start to analyze reasons behind those actions, which potentially builds on M-PSTs' analytical skills relating to teachers' knowledge of students' underdeveloped conceptions, patterns of student errors, and logical sequences of content. Such recognition and analysis potentially help them to make instructional decisions in their future classes. However, enhancing M-PSTs' conceptions of MKT is a complex task and does not happen instantly after watching and analyzing one or two video lessons. M-PSTs might take a long time to build on the following two conceptions: (a) SMK is not limited to CCK, and (b) PCK also includes content knowledge. Future researchers should consider studying PSTs' conceptions of KCS, KCT, and SCK through an extended (e.g., a semester) series of video analysis activities similar to those described in this study. Therefore, it is recommended that MKT framework should be used as a tool to enhance M-PSTs' awareness and reflective thinking in and across different dimensions of teacher knowledge in relation to each other within and beyond the framework that may affect their future practices in the classrooms.

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Author Information

Bima Sapkota

https://orcid.org/0000-0003-4992-2666

The University of Texas Rio Grande Valley

1201 W University Drive, EMAGC 3.720

Edinburg, Texas, 78539, USA

Contact e-mail: *bima.sapkota@utrgv.edu*

Appendix A. Prompts and Questions for Analysis of and Discussion on the Video Lesson

Day	Activities	Prompts and Questions
1	Reflection on the Video Lesson (Day 1 Reflection)	• What are your initial thoughts after watching the video? • Which teaching strategies did you think were particularly effective? Is there anything that you would do differently? • What did you learn about mathematics teaching and learning from this video?
2	Analysis of Day 1 Reflection Papers (Day 2 Analysis)	• Please analyze your Day 1 reflection paper using the MKT framework. Be sure to answer the following questions: (a) Do you see any examples of the MKT framework domains in your reflections of the video-lesson? (b) Which domains are not mentioned in your reflection paper? Explain your reasoning briefly.

2	Discussion on the Video Analysis and the MKT framework (Day 2 Discussion)	• Talk about the domains of the MKT framework that you addressed in your Day 1 reflection papers.
		• Talk about the domains of the MKT framework that you did not mention in your reflection paper.
		• What did you learn from this experience when analyzing your reflection paper using the MKT framework?
		• What are some challenges that you found when analyzing your reflection paper using this framework?
		• Reflecting on your experiences with courses in your program, describe one or more experiences that correspond to one of the categories of the MKT framework.
		    o Why do you think these experiences correspond to this domain?
		    o Reflect on the domains of the MKT framework that you have had both the most and least opportunities to engage with in your program.

3	Analysis of the Video Lesson (Day 3 Analysis)	• Watch the video and describe the teacher's and students' actions (e.g., asking a question, using manipulatives) that seem important to you for teaching and learning mathematics. You can use the domains of the Mathematical Knowledge for Teaching (MKT) framework to analyze those actions. You may pause the video or use the transcript if you want to mention specific actions (e.g., quoting the teacher's question) from the video.

Appendix B. Codes Emerged from M-PSTs' Descriptions of Classroom Events in the Video Lesson

Participants	Classroom events attended to in the beginning of the video analysis activities	Classroom events attended to in the conclusion of the video analysis activities
Mitchell	• Teacher direction vs teacher scaffolding- generic (1) • Student engagement - generic (1) • Student questioning- generic (1) • Instructional materials- generic (2) • Student thinking- generic (1) • Group work- generic (1) • Teacher directions- generic (1) • Problem solving strategy- specific (1)	• Teacher questioning- generic (1) • Student engagement- generic (1) • Group work- generic (1) • Teacher directions vs teacher scaffolding- generic (1) • Review students' pre-knowledge- specific (1) • Teacher's knowledge of exponent- specific (1) • Problem solving strategies- specific (1) • Building students' conceptual understanding- specific (1) • Teacher solving problem accurately using different strategies- specific (1) • Teacher explaining mathematical concept accurately- specific (1) • Student thinking- specific (1) • Techer explaining mathematical terminologies- specific (1) • Teacher directions vs teacher scaffolding- specific (1) • Individual work- specific (1) • Teaching objectives – specific (1)
Seth	• Classroom management- generic (2) • Student engagement- generic (1) • Teacher questioning- generic (2) • Wait time- generic (2) • Student thinking- generic (2) • Teacher questioning- generic (1) • Student engagement- generic (1) • Group work- generic (1)	• Review of students' pre-knowledge- specific (1) • Use of instructional materials- specific (1) • Use of various mathematical examples- specific (1) • Student thinking- specific (1)

	• Individual work- generic (1) • Teacher directions vs teacher scaffolding- generic (1) • Mathematical examples- specific (1)	
Jeremy	• Classroom management- generic (1) • Individual work- generic (1) • Teacher questioning- generic (2) • Group work- generic (2) • Wait time- generic (1) • Teacher direction vs teacher scaffolding- generic (1) • Group work- generic (1) • Instructional materials- specific (1) • Teacher questioning- specific (1) • Instructional materials- specific (1) • Group work- specific (1)	• Student questioning- generic (1) • Revision of students' prior knowledge- specific (1) • Group work- specific (1) • Teacher questioning- specific (1) • Teacher directions- specific (1) • Group work- specific (2) • Individual work- specific (1) • Wait time- specific (1)
Jonah	• Classroom management- generic (1) • Review prior knowledge- generic (1) • Wait time- generic (1) • Student engagement- generic (1) • Teacher directions vs scaffolding- generic (1) • Teacher direction vs scaffolding- specific (2)	• Student questioning- generic (1) • Teachers' knowledge of content- specific (1) • Teacher's knowledge of various representations of mathematical concept- specific (1) • Individual and group work- specific (1) • Teacher's knowledge of the concepts that are required for learning the concept further- specific (1) • Teacher's knowledge of students' pre-knowledge- specific (1)

Note: The numbers in the parenthesis indicate the number of times that the participants attended to a particular classroom event