Bibliometrics Analysis of Research on Argumentation in Mathematics Education

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Article Info	Abstract
	This study is based on the bibliometrics analysis of research publications that focus on highlighting the argumentation on mathematics education. The main objective of this study is to analyze the distribution of research results, the main journals for publication, and the most productive authors, the pattern of authorship and the pattern of citation. Data was collected from Scopus database and was analyzed using VOSviewer software. A total of 173 articles published from 1995 to December 2021 were retrieved and perused. Graphs, tables, charts, and maps have been used to analyze and present collected data. The analysis results show that argumentation in mathematics education has been studied frequently with the concepts of proof, reasoning, teacher education, and geometry. Journal of Mathematical Behavior and Educational Studies in Mathematics have been the top journals, and the University of Rutgers has been the most influential institution, and the United States has had a significant prevalence in this field. The results of this study provide insights for researchers interested in mathematics education in this field, suggesting that they can explore various alternatives to the major research trends and the influential articles mentioned. Based on the results, recommendations for future studies are provided.

Introduction

Over the past few decades, research into argumentation in mathematics education has been conducted in a large number of ways and published in various scientific journals. This indicates that the subject of argumentation has become a focus of attention in the research and curriculum of mathematics education (Gomez Marchant, 2021; Zhou et al., 2021; Francisco, 2022). For example, the Common Core State Standards for Mathematical Practice (CCSSMP) in the United States include argumentation standards. CCSSMP requires students to "construct viable arguments and critique the reasoning of others" (*National Governors Association Center*, [NGAC], 2010).

In addition, the results of accumulated research show that participation in argumentative activities is effective in promoting student learning through the development of social and cognitive processes; which encourage students to explore, confront, and evaluate alternative positions, voice support or objections, and justify different ideas and

hypotheses; promotes meaningful understanding and deep thinking (Ayalon & Even, 2016; Dovigo, 2016). Engaging students in mathematical arguments supports the development of language and conceptual understanding because students use verbal and written language in coordination with mathematical representations to notice patterns, make claims, create, develop, construct and revise explanations, and communicate their mathematical knowledge (Stylianides, 2018; Kazemi et al., 2021).

Literature Review

Argumentation is defined as the process of constructing arguments within the realms of discussion, dialogue, or persuasion. An argument, on the other hand, is the result or outcome of engaging in such activities (Kuhn & Udell, 2007; Macagno et al., 2014; Schwarz & Shahar, 2017). For the purpose of this study, argumentation is specifically defined as the process of conveying opinions, ideas, or concepts by presenting arguments in the form of statements that serve to support other statements, namely conclusions. Argumentation entails making a claim, providing evidence to substantiate that claim, and evaluating the said evidence in order to determine the validity of the claim (Ayalon & Hershkowitz, 2018). An argument, therefore, refers to a collection of statements that emerges as a result of argumentation. These arguments encompass statements utilized to express an individual's position on a particular issue with the intention of reinforcing a conclusion. Consequently, an argument can be regarded as a constituent element of argumentation, comprising a claim (conclusion or solution) supported by principles or theories, evidence, and counter-arguments that address potential opposing arguments (Jonassen, 2010), as well as other statements in the form of evidence, support, and counter-arguments. According to Govier (2013), when someone presents an argument, they provide reasons and evidence in an attempt to persuade others of the correctness of their beliefs.

In the field of mathematics education, the term "argumentation" encompasses two distinct concepts. Firstly, it denotes the act of presenting mathematical arguments by both students and teachers within a classroom environment. Secondly, it refers to arguments put forward by researchers in mathematics education, concerning the nature of mathematical learning and the effectiveness of teaching mathematics in different contexts (Sriraman & Umland, 2020). According to Sriraman & Umland (2020), mathematical argumentation in the classroom involves the presentation of a logical sequence of reasoning intended to establish the validity of a mathematical outcome. Within the realm of mathematics education research, numerous scholars have underscored the importance of incorporating activities centered around argumentation in the classroom. Such activities serve as a means of enhancing students' comprehension of mathematical concepts and their capacity for mathematical reasoning (Erkek & Bostan, 2019).

Mathematical argumentation is a specific form of discourse that involves the utilization of justification, correlation, and the incorporation of ideas (Ibraim & Justi, 2016; Uygun & Guner, 2019). This type of conversation is directed towards establishing the accuracy of mathematical statements (Knudsen et al., 2014; Rumsey & Langrall, 2016). It can be defined as a succession of statements and rationales intended to substantiate the validity of a proposition (Cardetti & LeMay, 2018).

Mathematical argumentation encompasses a diverse range of activities, including conjecturing, exemplification, thought experiments, representing mathematical ideas, adopting alternative perspectives, analyzing, and revising (Staples & Newton, 2016). It necessitates students to engage in dialogue by providing their own arguments and counterarguments, constructing explanations, posing inquiries, and potentially challenging the arguments put forth by others. Mathematical argumentation entails the process of constructing persuasive arguments to demonstrate or elucidate the validity of mathematical statements or solutions to mathematical problems. As it is inherently a collaborative endeavor, it is imperative for researchers to closely observe and analyze the unfolding discourse to gain insights into the nature of students' argumentation.

Furthermore, articles published in scientific journals can serve as sources of information and data for research such as systematic reviews (e.g. Campbell, 2019), content analysis of articles (e.g. Kartika et al., 2021), and bibliometrics analysis. In this study, we decided to examines the analytical methods of bibliometrics to identify the bibliometric data in articles published in scientific journals on the argumentation research in mathematics education.

The term bibliometrics refers to the application of quantitative methods to all types of resources in a research field (Pritchard, 1969). Bibliometric research includes the statistical study of published papers, which defines the pattern of publication authorship and citation in certain fields, and their evolution overtime (Mathankar, 2018; Julius et al., 2021; Thi-Trinh et al., 2021). This implementation enables data analysis from citation indices to assess the reputation and influence of specific articles, authors and research publications (Julius et al., 2021). Bibliometric data include descriptive metadata that characterize the source document, such as authors, countries, institutions, keywords, language, source of publication, publication year, cited reference, source of reference and subject category (Drijvers et al., 2020).

Bibliometric analysis has been widely used to analyze scientific evolution in the fields of mathematics education such as mathematics education research (Julius et al., 2021), mathematics education research in the case of instrumental orchestration (Drijvers et al., 2020), realistic mathematics education (Phan et al., 2022), teaching and learning of ordinary differential equations (Lozada et al., 2021), mathematics anxiety (Ersozlu & Karakus, 2019), artificial intelligence in mathematics education (Hwang & Tu, 2021), application of ICT in mathematics education (Phuong et al., 2022), and mathematics problem solving in elementary education (Suseelan et al., 2022). However, the application of these bibliometric tools to argumentation research in mathematics education has not been published to date.

Purpose of the Study

This bibliometric analysis of argumentation research in mathematics education aims to describe and discover current trends, topics, and scientometric characteristics in this body of literature, providing a high-level overview of scientific literature and provides insights into future directions of argumentation in mathematics education. In particular, three following research questions would be examined:

1) Which international cooperation among countries have had the greatest impact on the argumentation in

mathematics education literature?

2) What are the most important source journals, the most effective scholars, and the most influential articles on the argumentation in mathematics education literature?

3) What are the most popular research topics and trends based on word analysis of titles, abstracts, and keywords on the argumentation in mathematics education literature?

Method

This study used the method of a common scientific mapping workflow based on the following five rigorous steps: 1) Design of the study, 2) Collection of data, 3) Analysis of data, 4) Visualization of data and 5) Interpretation (Börner et al., 2005; Zupic & Cater, 2015).

Study Design

A bibliometric analysis aims to create a quantitative study using statistical techniques to assess many aspects of a given body of bibliographic data, including journals, research institutions, geographic location, and other features (Narin & Hamilton, 1996). Consequently, the study also included quantitative research with meta-analysis design because the objective of this study is to identify bibliometric data from scientific papers on argumentation in mathematics education that are listed in the Scopus database.

Data Collection

This stage is divided into four sub-stages, namely data collection, data filtering, data transformation and data cleaning. In order to collect data, the study used the Scopus database as provided by Drijvers et al. (2020); Julius et al. (2021); Phan et al. (2022) and Chin et al. (2022). The selection of the Scopus bibliographic database was made due to its extensive coverage of materials, surpassing other databases by 70% in terms of sources (Ha et al., 2020; Mongeon et al., 2016). As a result, the integrity of research data is guaranteed. We searched for articles published between 1995 and 2021 (excluding conference papers, books, and their chapters). We chose the beginning of the year of search based on the fact that Gotz Krummheuer published his first scientific paper in mathematics argumentation literature in 1995. In the search for articles focusing on mathematical argumentation, we decided to include articles using the words "argumentation", "argument" or "argumentative" in titles, abstracts or keywords. The justification of this approach is to include articles in which argumentation is the main focus of the concept.

Data Filtering

We filtered the data by selecting titles, abstracts, and keywords, and decided, as a quality criterion, to include only academic journals in the 2021 Scimago Journal Ranking List, writings in English and discussing the topic of argumentation from the point of view of mathematics education. We also removed articles that were not relevant to research on argumentation in mathematics education (such as applied mathematics, pure mathematics) and

articles that were not directly related to research objectives.

Data Transformation

The search outcomes were saved in CSV format to including all the crucial papers information: (1) citation information (authors' name, document titles, years, etc.), (2) bibliographical information (affiliations, publisher, etc.), and (3) abstract and keyword (abstract, author keywords, index keywords).

Data Cleaning

Once MS Excel was used to import from a text, CSV file a data cleaning process was performed to verify incomplete or incorrect input entries, correcting some duplicate and inconsistent information in the collected data such as: source title, affiliations, etc. In this process, two specific steps have been adopted: i) verifying field (column) entries to ensure that important data are not missed, and ii) cross-checking the data in columns to ensure that the data content of the fields is consistent with the field title. If the incorrect or missing entry is identified, it is deleted accordingly (Shareefa & Moosa, 2020). For example, "ZDM - International Journal on Mathematics Education" and "ZDM - Mathematics Education" were corrected as one source title, "Graduate School of Education, Rutgers University" and "Rutgers University" were corrected as one affiliation, and University of Exeter, South Africa were corrected as in United Kingdom. Figure 1 shows the flow chart of data collection and search strategies in the Scopus database prior to further analysis.
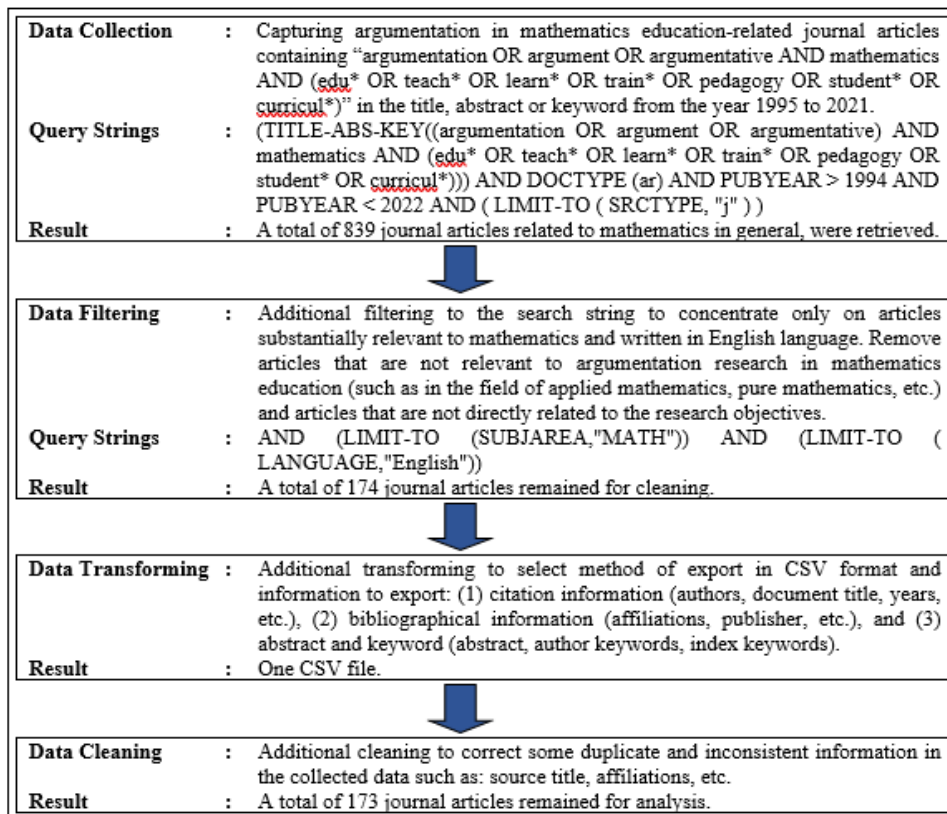
Data Collection	:	Capturing argumentation in mathematics education-related journal articles containing "argumentation OR argument OR argumentative AND mathematics AND (edu* OR teach* OR learn* OR train* OR pedagogy OR student* OR curricul*)" in the title, abstract or keyword from the year 1995 to 2021.
Query Strings	:	(TITLE-ABS-KEY((argumentation OR argument OR argumentative) AND mathematics AND (edu* OR teach* OR learn* OR train* OR pedagogy OR student* OR curricul*))) AND DOCTYPE (ar) AND PUBYEAR > 1994 AND PUBYEAR < 2022 AND (LIMIT-TO (SRCTYPE, "j"))
Result	:	A total of 839 journal articles related to mathematics in general, were retrieved.

Data Filtering	:	Additional filtering to the search string to concentrate only on articles substantially relevant to mathematics and written in English language. Remove articles that are not relevant to argumentation research in mathematics education (such as in the field of applied mathematics, pure mathematics, etc.) and articles that are not directly related to the research objectives.
Query Strings	:	AND (LIMIT-TO (SUBJAREA,"MATH")) AND (LIMIT-TO (LANGUAGE,"English"))
Result	:	A total of 174 journal articles remained for cleaning.

Data Transforming	:	Additional transforming to select method of export in CSV format and information to export: (1) citation information (authors, document title, years, etc.), (2) bibliographical information (affiliations, publisher, etc.), and (3) abstract and keyword (abstract, author keywords, index keywords).
Result	:	One CSV file.

Data Cleaning	:	Additional cleaning to correct some duplicate and inconsistent information in the collected data such as: source title, affiliations, etc.
Result	:	A total of 173 journal articles remained for analysis.

Figure 1. Flowchart of Data Collection

Data Analysis and Visualization

Various analytical techniques were used to extract the information from the series of publications. General information on the collection of publications has been summarized and the annual number of publications is analyzed to determine trends in the development of the research field. The contributions of each country were analyzed on the basis of the number of articles and citations to determine the most productive. In this paper, the VOSviewer software was used to collect, analyze and visualize bibliographic data using the supported CSV file type. The bibliographic coupling of sources, authors, countries, institutions, publications and the co-occurrences of author keywords has been analyzed and visualized.

Results

General Information and Growth Trend

The most important information from the collection on argumentation in mathematics education is shown in Table 1. Although authors listed publications from 1995 to 2021, the first published papers in this area appeared in 1996. Between 1996 and 2021, the authors published 173 articles in 26 Scopus-indexed journals.

Table 1. The Collection's Primary Information

Description	Results
PRIMARY DATA INFORMATION	
Interval	1996:2021
Sources (Journals)	26
Documents (Articles)	173
Distribution by Country	39
Contribution by Institutions	293
AUTHORS	
Authors	320
Authors of multi-authored documents	269
Authors of single-authored documents	51
COLABORATION BETWEEN AUTHORS	
Single-authored documents	14
Authors per Document	1.85
Documents per Author	0.54

A total of 320 authors were published in 173 publications (equivalent to 1.85 authors per document), in which the number of authors of multi-author documents accounted for most of the collection with 269 (84.1%). The number of authors of single-authored documents represented only a 15.9% ratio (51 researchers). These researchers published 14 single-authored documents (8.1% of publications) in the collection.

Information on the annual volume of publications on argumentation in mathematics education studies between

1996 and 2021 at an interval of 31 years is shown in Figure 2. The cluster column shows the annual increase in the publication, and the solid line represents the linear (total number of publications). The publication volumes for the first decade of the selected period (1996-2006) are relatively low and less than 5 articles are published annually. The number of publications reported to have increased significantly in 2007, reflecting the growing interest in research. In addition, this could also show other things, such as the growth of education and mathematics due to more publishing options, a broader number of active researchers and an increase in the network of collaborators. These numbers fluctuate over time. The cumulative data show a strong general growth trend, especially in the last five years (2017-2021) of the total number of 76 publications intervals. The number of publications is expected to increase further in accordance with this growth trend.
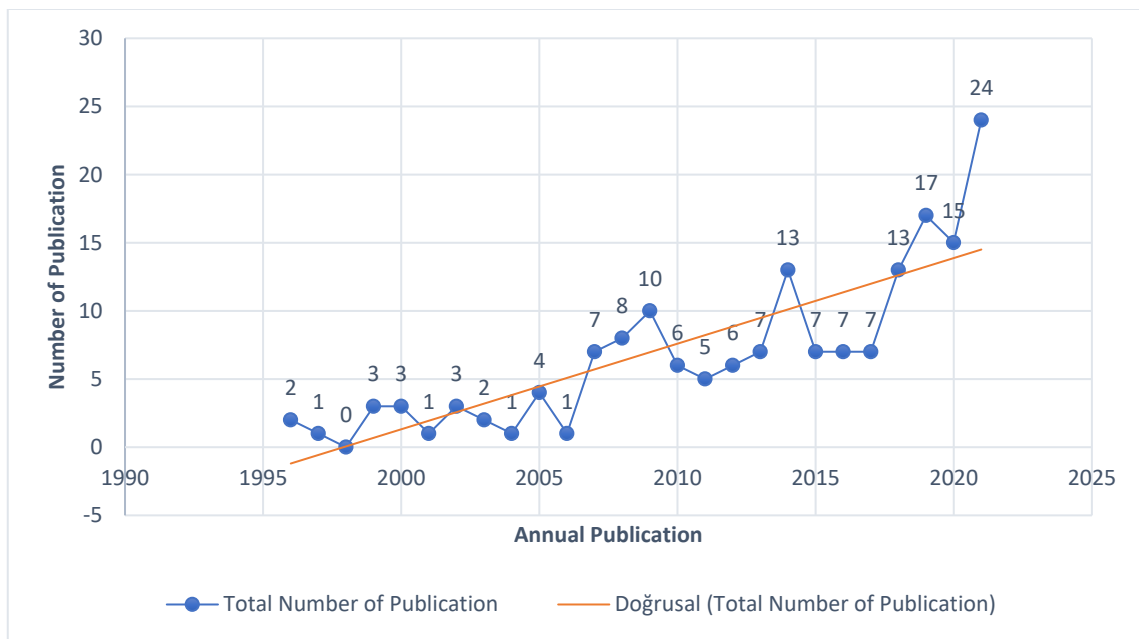

Figure 2. Distribution of Articles

The statistics on the number of citations for publications related to research into argumentation in mathematics education are shown in Table 2. The number of articles not cited in the document was 22 (12.7%), and there are 93 articles with citations from 1 to 10 (53.8%). The number of articles with more than 55 citations is 14 (8.1%), of which only six articles have more than 100 citations.

Table 2. Number of Citations in the Collection of Publications

NC	ND	%
>100	6	3.5
55 to 100	8	4.6
21 to 54	16	9.2
11 to 20	28	16.2
1 to 10	93	53.8
0	22	12.7

Notes. *NC*= number of citations, *ND*= number of documents, *%*=percentage

Distribution by Country

According to Scopus data, authors from 39 countries/regions participated in the study. However, only countries with at least five publications were included in the analysis. Of the 39 countries, 10 met the threshold. The figure 3 shows the contributions of countries and the information on the top 10 countries with the largest number of publications in the field of argumentation in mathematics education. Most of the publications were contributed by authors from the United States, with 84 studies (53.2%) published. The other countries on this list contribute significantly less than the first-rank country: Germany (14 papers, 8.9%) and Turkey (12 papers, 7.6%); seven countries placed at the bottom of the rank with five to ten articles: United Kingdom, Israel, Chile, Australia, Spain, Greece, New Zealand.

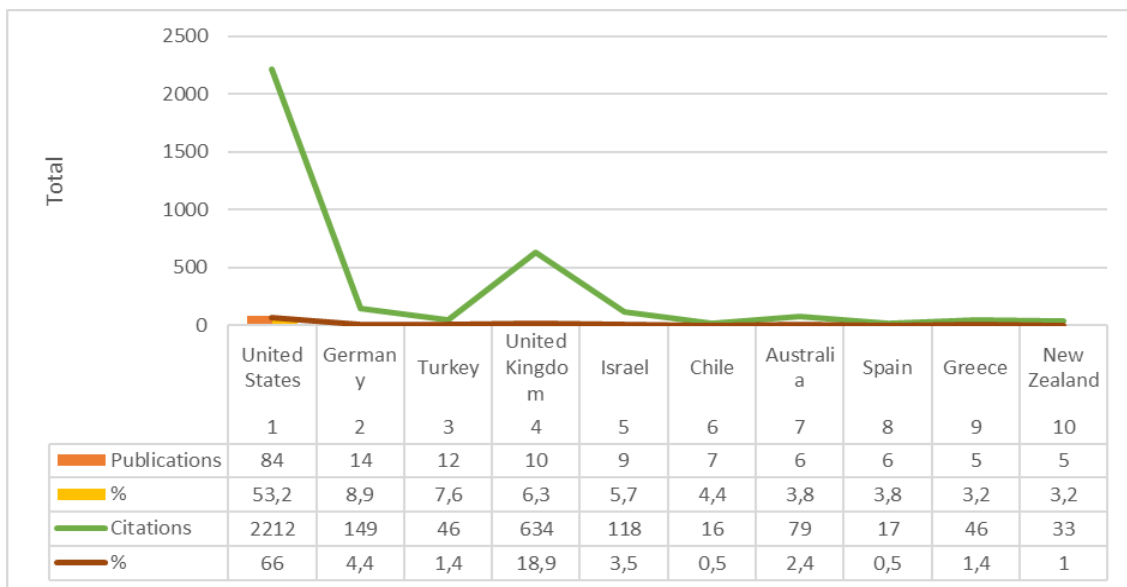|	United States	Germany	Turkey	United Kingdom	Israel	Chile	Australia	Spain	Greece	New Zealand
	1	2	3	4	5	6	7	8	9	10
Publications	84	14	12	10	9	7	6	6	5	5
%	53,2	8,9	7,6	6,3	5,7	4,4	3,8	3,8	3,2	3,2
Citations	2212	149	46	634	118	16	79	17	46	33
%	66	4,4	1,4	18,9	3,5	0,5	2,4	0,5	1,4	1

Figure 3. Top 10 Countries with the Most Publications

As far as the total number of citations per country is concerned, the United States is still the country with the highest number of citations with 2212 citations (about 66.0%). Interestingly, despite The United Kingdom (UK) is the fourth most active country with 10 (6.3%) total of publication, the total number of citations (634, 18.9%) contributed significantly more than the country in the second (149, 4.4 %) and the third (46, 1.4%) followed by Israel with 118 citations (3.5%) and Australia with 79 citations (2.4%). In the other countries, the number of citations varies between 16 and 46.

Contribution by Institutions

According to the Scopus database obtained, 173 papers are published by 293 different affiliates. However, only organizations with at least two publications were included in the analysis. Of the 293 organizations, 4 met the threshold. The fourth most productive institution is shown in Figure 4 on the basis of the number of publications. As can be seen, most of the affiliations listed come from the United States. Only one affiliation come from Israel. The University of Rutgers has 7 papers (4.0%) and ranks first. In the second rank, the University of Idaho (4,2.3%),

followed by the Weizmann Institute of Sciences, with three papers from Israel, and the University of Purdue with two papers. Interestingly, the University of Purdue has only two papers, but 92 outstanding citations. This is due to the fact that this institution has contributed to the two articles with the highest number of citations in the collection: (Wood, 1999) with 84 citations and (Max & Welder, 2020) with 8 citations.
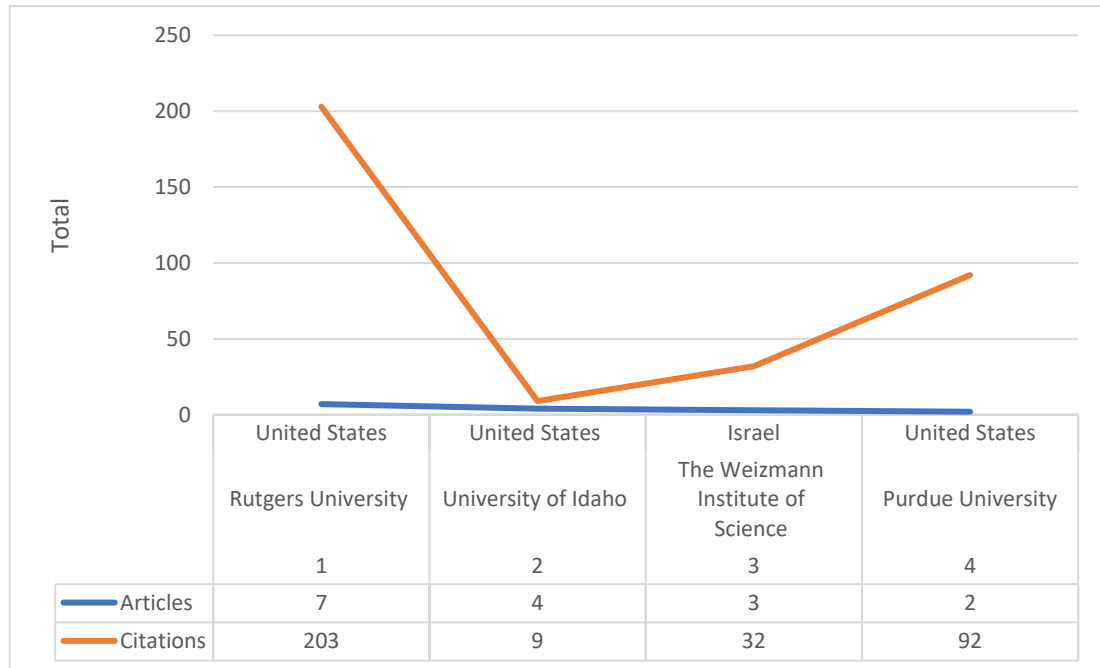

	United States	United States	Israel	United States
	Rutgers University	University of Idaho	The Weizmann Institute of Science	Purdue University
	1	2	3	4
Articles	7	4	3	2
Citations	203	9	32	92

Figure 4. The Top Four Most Productive Publishing Institutions According to the Total Number of Publications.

The international cooperation network (at least five documents) in the field of arguments in mathematics education is shown in Figure 5. The size of the node indicates the number of publications, and the thickness of the line between the node indicates the strength of the cooperation. According to the Scopus database, 39 countries have authors participating in research and publication in this field. However, international cooperation for research is not very significant, mainly in the United States with other countries such as Turkey, New Zealand, the United Kingdom and Australia.

Figure 5. International Cooperation Network

Contribution by Journals

As shown in Table 1, 173 papers have been published in 26 different sources. However, only sources with at least five publications were included in this study. Of the 26 sources, 10 met the threshold. The top 10 journals that publish papers in the field of argumentation in mathematics education are listed in Table 3. These ten journals

therefore published 146 papers, accounting for 84.4% of the collection with a citation of 3227, or 92.7% of the total citation. The Journal of Mathematics Behavior has published the largest number of papers on this topic, with 30 papers. After that, there were 22 papers on educational studies in mathematics and 19 papers on the International Journal of Science and Mathematics Education. The following seven positions on this list contain between six and eighteen papers on this topic. Nevertheless, Journal for Research in Mathematics Education published only nine papers, but 1581 was outstanding cited. This is due to the fact that the journal published the first article in this field in 1996 and received the highest number of citations in the collection (823 citations in (Yackel & Cobb, 1996)).

Most of these journals are highly ranked in Scopus Q1 journals rankings with 8 journals. Only one journal was ranked Q2 and one journal was ranked Q3. The Educational Studies in Mathematics is ranked by SJR as the best-performing journal (1.54); it ranks 74/1381 among Scopus's educational topics. The second rank was ZDM – Mathematics Education, with SJR 1.37, with a rating of 95/1381. The third rank is the Journal of Mathematics Teacher Education, with SJR 1.29 rating of 111/1381. These journals come from four different publishers: Springer Nature (5), Taylor & Francis (3), Elsevier (1), and National Council of Teachers of Mathematics (NCTM) (1).

Table 3. List of the Most Published Sources of Research on Argumentation in Mathematics Education

No.	Source	Publisher	NP	NC	Scopus Quartile[*]	SJR 2021[*]
1	Journal of Mathematical Behavior	Elsevier Inc.	30	346	Q1	0.95
2	Educational Studies in Mathematics	Springer Nature	22	686	Q1	1.54
3	International Journal of Science and Mathematics Education	Springer Nature	19	148	Q1	1.15
4	International Journal of Mathematical Education in Science and Technology	Taylor & Francis	18	107	Q2	0.48
5	ZDM – Mathematics Education	Springer Nature	16	126	Q1	1.37
6	Mathematics Education Research Journal	Springer Nature	10	95	Q1	0.78
7	Journal for Research in Mathematics Education	NCTM	9	1581	Q1	1.07

No.	Source	Publisher	NP	NC	Scopus Quartile*	SJR 2021*
8	PRIMUS	Taylor & Francis	9	18	Q3	0.32
9	Journal of Mathematics Teacher Education	Springer Nature	7	65	Q1	1.29
10	Research in Mathematics Education	Taylor & Francis	6	55	Q1	1.01

Notes. *NP*=number of publications, *NC*=number of citations, *Reference information from
https://www.scimagojr.com/ on September 01, 2022

The bibliographic coupling between these top 10 journals is illustrated in Figure 6, which shows two different groups. The first group consists of eight journals: Journal of Mathematical Behavior, Educational Studies in Mathematics, International Journal of Science and Mathematical Education, International Journal of Mathematical Education in Science and Technology, ZDM - Mathematics Education, Mathematics Education Research Journal, Journal for Research in Mathematics Education, and Journal of Mathematics Teacher Education. The final two journals are PRIMUS and Research in Mathematics Education. They are grouped together in the second group.
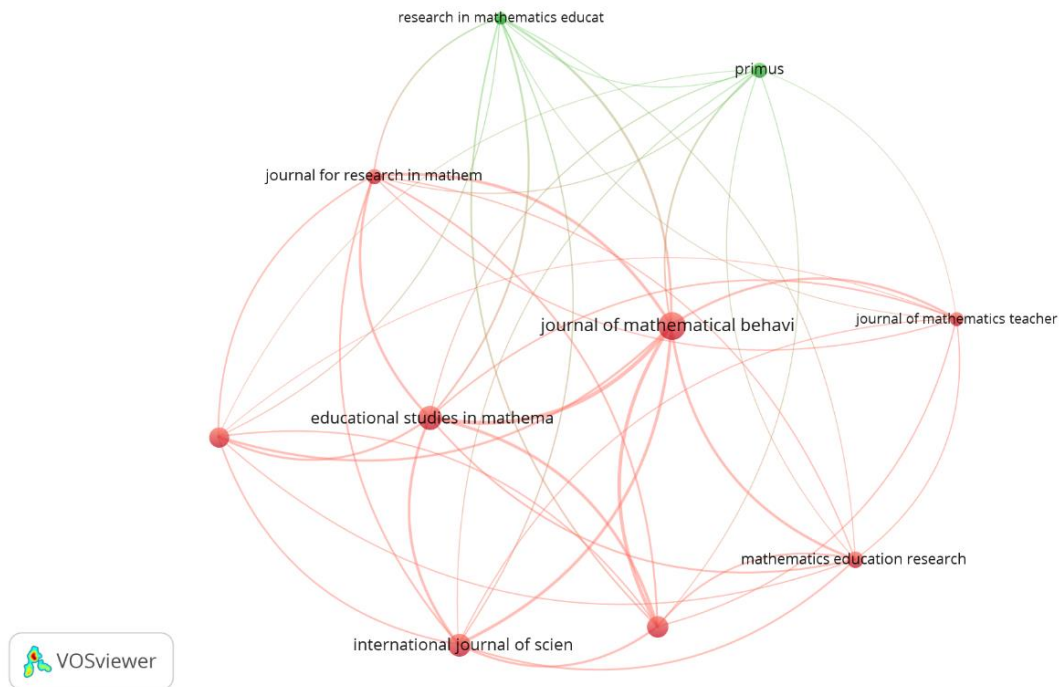

Figure 6. Bibliographic Coupling of the Sources (Network Visualization)

Contribution by Authors

The most productive authors, based on the number of publications, citations, and h-indexes, are listed in Table 4. Each author had at least three publications that were included in the analysis. Of the 320 authors, 5 met the thresholds. Most of the authors come from U.S. institutions. Only one author comes from the German institution.

The number of papers by the first authors is not too different and not too high, with only 3–6 papers. The h-indexes of the first and highest number of authors (only for the articles in the research collection) are Weber, K. of the University of Rutgers with 20 h-indexes and 6 articles. In the second and third categories, there were 5 and 4 papers, namely Yopp, D.A. from Idaho University and Conner, A.M. from Georgia State University. Two authors with three papers were fourth among the authors: Smith, R.C. of the University of Georgia and Conner, A.M. of the University of Georgia. Weber, K. (195 citations) is the most cited, with 86 in the document (Weber, 2008). The second and third ranks are Smith R.C. with 110 citations and Conner, A.M. with 91 citations. These two authors were co-authors of a paper with a large number of citations (there are 62 in the paper (Conner et al., 2014) until research was conducted). The other two authors were Krummheuer, G. of the University of Goethe, with 90 citations, and Yopp, D.A. of the University of Idaho, with 11 citations.

Table 4. The Four Most Productive Authors

Rank	Author	CA	Country	NP	NC	H-Index
1	Weber, Keith	Rutgers University	United States	6	195	20
2	Yopp, David A.	University of Idaho	United States	5	11	8
3	Krummheuer, G.	Goethe University	Germany	4	90	6
4	Smith, Ryan C.	University of Georgia	United States	3	110	4
4	Conner, Anna M.	University of Georgia	United States	3	91	7

Notes. *CA*=current affiliation, *NP*=number of publications, *NC*=number of citations

Figure 6 shows the network of co-authors of at least three publications in the field of argumentation in mathematics education. Authors are represented by nodes. The size of the node shows the number of publications, and the thickness of the line between the node shows the strength of the collaboration (typically a number of publications in which they are co-authors). It can be seen that collaboration between researchers on this subject is not strong and is conducted mainly in small groups of two to four researchers. These groups have little cooperation. It is shown that the figure has two groups of authors.
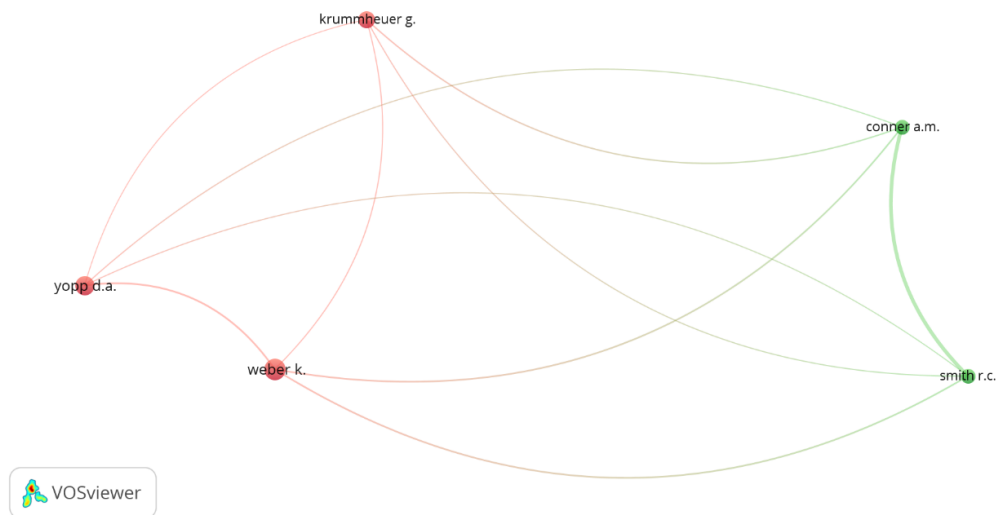

Figure 7. Bibliographic Coupling of the Authors (Network Visualization)

The Most Influential Articles

The Table 5 shows information on the most influential studies in this field. Only publications with at least 70 citations were included in the analysis. Of the 173 documents, 10 met the threshold. The highest total citation (823 recorded) was written by two authors, Yackel, E. and Cobb, P. These article about sociomathematical norms, argumentation, and autonomy in mathematics published in Journal for Research in Mathematics Education (ranked 157/1381 in the field of Education by Scimagojr) in 2021. The author of the article suggests that sociomathematical norms are interactively constructed and formed to illustrate how these norms regulate mathematical arguments and influence learning opportunities for both students and teachers (Yackel & Cobb, 1996). With 235 citations, the article by Healy L. and Hoyles C. on a study of proof conceptions in algebra was published in Journal for Research in Mathematics Education is the second on this list. Although the title does not include the word "argument" or "argument", the abstract explains the term. The author of the article suggests that empirical argument predominated in students' own proof constructions (Healy & Hoyles, 2000). Some other articles that study argumentation and proof are (Selden & Selden, 2003), (Stylianides & Stylianides, 2009) and (Weber, 2008). Some other topics that are also of interest to researchers are the mathematics and science integration argument (Furner & Kumar, 2007); Modelling mathematical argumentation: The importance of qualification (Inglis et al., 2007); Argumentation using dynamic geometry software (Jones, 2000); The arguments for using history in the teaching and learning of mathematics (Jankvist, 2009); Creating a context for argument in mathematics class (Wood, 1999).

Table 5. Overview of the Top 10 Most Cited Papers in the Publication Collection

Rank	Article	First Author Institution/Country	NC
1	Yackel & Cobb (1996)	Purdue University, United States	823
2	Healy & Hoyles (2000)	University of London, United Kingdom	235
3	Selden & Selden (2003)	Tennessee Technological University, United States	170
4	Furner & Kumar (2007)	Florida Atlantic University, United States	117
5	Inglis et al. (2007)	University of Nottingham, United Kingdom	109
6	Jones (2000)	University of Southampton, United Kingdom	101
7	Stylianides & Stylianides (2009)	University of Pittsburgh, United States	94
8	Jankvist (2009)	Roskilde University, Denmark	86
8	Weber (2008)	Rutgers University, United States	86
9	Wood (1999)	Purdue University, United States	84

Notes. NC=Number of Citations

Keywords and Terms Analysis

Figure 8 shows a keyword showing trends in research on argumentation in mathematics education. Only keywords that occur at least five times have been included in the analysis. Of the 584 terms, 13 keywords met the threshold. Keywords that are highly related are grouped in similar colors, and the links between keywords represent when these keywords appear together in publications, the size of the keyword button indicates the number of times the keyword appears in the publications.
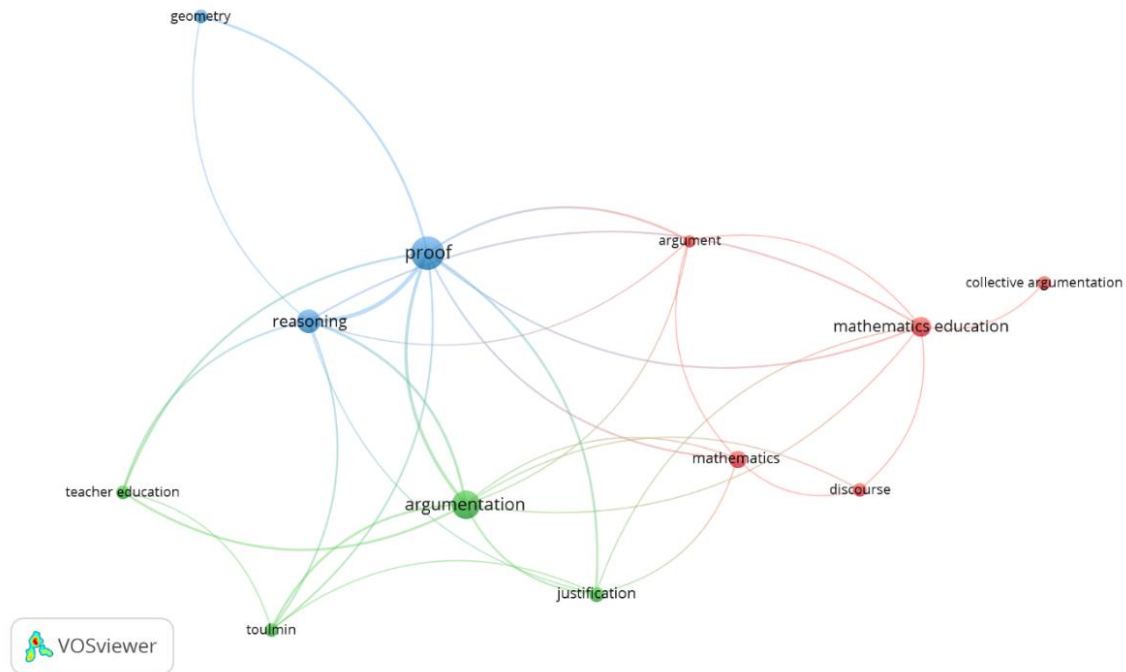

Figure 8. Keywords that Appear Most in Keywords of Publications

Analyzing the occurrence of keywords in argumentation research in mathematics education shows that authors are interested in many different directions of research, and the research trend with the most attention is proof (appearing in 33 studies-accounting for 22.9%) in this direction. There are some remarkable keywords: geometry and reasoning. The second keyword group is the research direction of argumentation, justification, teacher education and Toulmin. The last group of keywords is related to keywords such as argument, collective argumentation, and discourse.

Discussion

In this study, the bibliographic data from the Scopus database were used to draw a general picture of all publications that have been studied and discussed on argumentation in mathematics education over the last two decades (1996–2021). The first study of argumentation in mathematics education was conducted in 1996, with the first publication was done by authors from Purdue University and Vanderbilt University, including Erna Yackel and Paul Cobb. The publication attracted the interest of many researchers in the same field, as 823 citations were documented. Between 1996 and 2006, the number of publications in this field was relatively low, with 21

in total over five years (only 12.1% of the number of publications over the past two decades has been published). During the second half of the last five years (2017-2021), the number of studies on this topic increased dramatically, with 76 publications (43.9% of the total collection) (see Figure 2). This heightened scholarly interest can be attributed to the increasing emphasis placed on nurturing students' mathematical argumentation abilities within curricula across multiple nations (Kollar et al., 2014; Fukawa-Connelly & Silverman, 2015).

In the last two decades, half of the studies were published by U.S. researchers and groups, accounting for 48.6% of all publications. The United States is also the country with the highest total number of citations with 2212 citations (equivalent to 63.6%) (see Figure 3). There is no significant international cooperation for the conduct of studies; it is primarily American cooperation with other countries (see Figure 5). Therefore, the United States can be said to have played an important role in the establishment and continuation of scientific relations between countries. The role of the United States is also confirmed in most of the data of this study on the list of the four most productive institutions (see Figure 4), the list of the four most productive authors (see Table 4), and the list of the top ten papers with the greatest citation (see Table 5). Thus, it can be said that the United States is promoting research in the field of argumentation in mathematics education.

When analyzing the source of the publication, the authors found that researchers tend to publish in journals that are highly ranked in the Scopus journal rankings (see Table 3), the top ten journal of 9/10 is ranked in Q1 and Q2. This also shows the quality of research in this field. Data on the number of publications in the top four authors in this field indicate that research in the field of argumentation in mathematics education is one of many research directions, the number of publications of these authors was not too different and was not high (3–6 articles). Although the number of publications is not high, some authors have a significantly higher total number of citations than others because they have contributed to an article with a relatively high number of citations: Weber, K. (195 citations), Smith, R.C. (110 citations), Conner, A.M. (91 citations) (see Table 4).

Publications with the greatest influence in this area tended to link argumentation to proof. This is also consistent with the keyword analysis in Figure 8 shows that the research topics are not diverse, mainly focusing on several issues: proof, reasoning, teacher education, geometry, etc. These topics are divided into three major groups, some of which are related to each other.

Conclusion

This study uses bibliographic data from the Scopus database to investigate the development of publications in the field of argumentation in mathematics education published over the past two decades. The main findings of the study are as follows: 1) Although this is a possible direction of research, the number of studies on this subject is low. 173 studies were published in scientific journals between 1996 and 2021. Publications have appeared mainly in recent years, particularly in 2017 and 2021. 2) Although publications are published mainly in sources with high citation indexes, studies on this subject have not received sufficient attention, and the average citation number (23.04) is not large. 3) The United States is the country with the greatest influence in this field of research; the affiliations and authors have the greatest influence, almost all of which come from this country. 4) Research

cooperation in this field is not strong, and cooperation between the United States and other countries such as Germany, the United Kingdom, Israel, and New Zealand is mainly. 5) The quality of publications in this field is relatively high when many publications are published in high index journals. 6) Some of the main research directions on this subject are proof, reasoning, teacher education and geometry.

It is expected that the study of argumentation in mathematics education will increase in the future. In particular, with regard to the research trend in teacher professional development and proof, with more sub-areas and assessment methods. However, the knowledge of argumentation can be effectively transferred among various disciplines and educational levels (Noroozi et al., 2018). As a result, researchers interested in mathematics education in this field can look for different alternatives to the major research trends and the above-mentioned influential articles. Scholars also need to further strengthen international cooperation to improve the quality of future research.

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